Moce i Kompensacja w Obwodach z Niesinusoidalnymi Przebiegami Prądu i Napięcia

Cz. 1. Moce w obwodach jednofazowych

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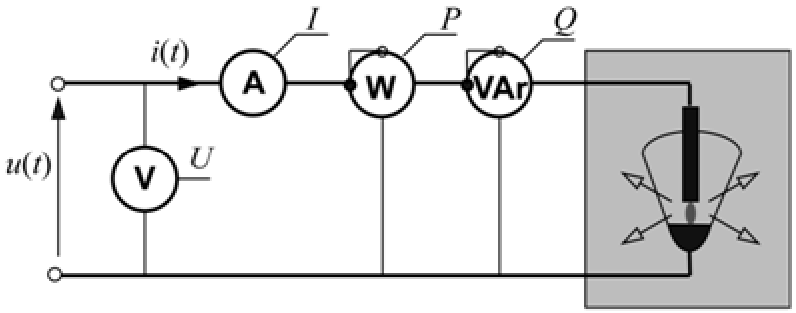
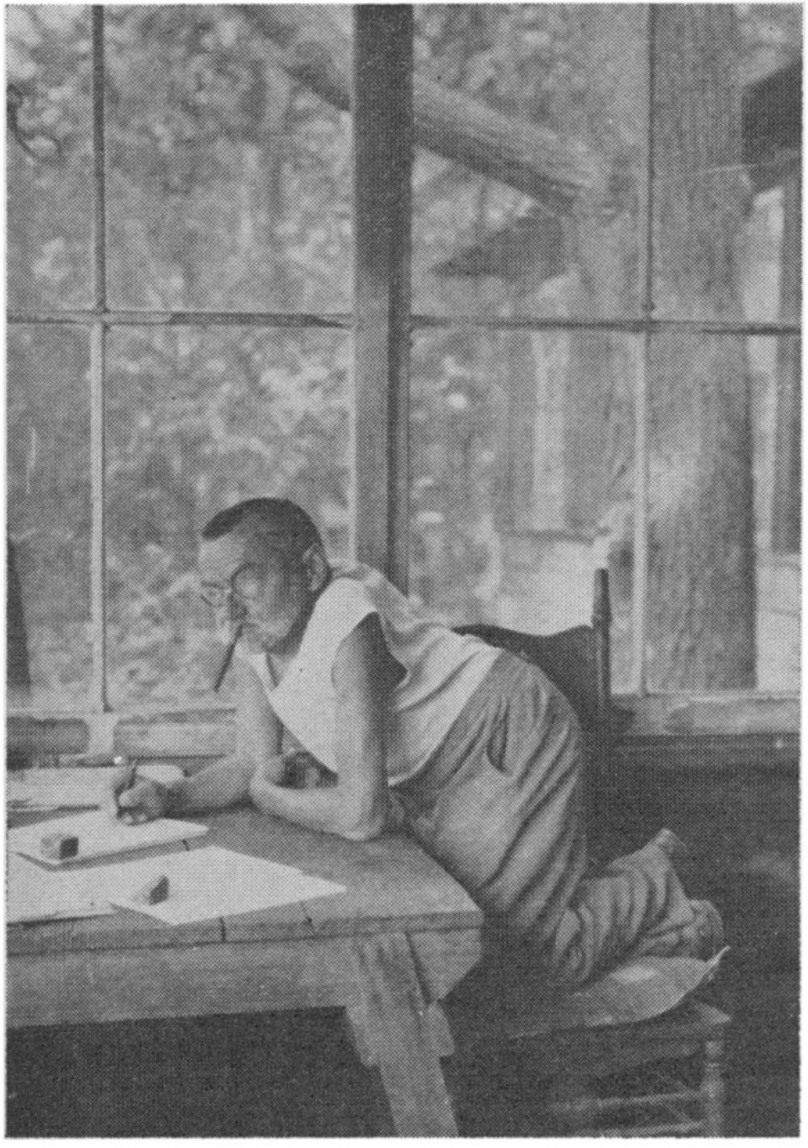
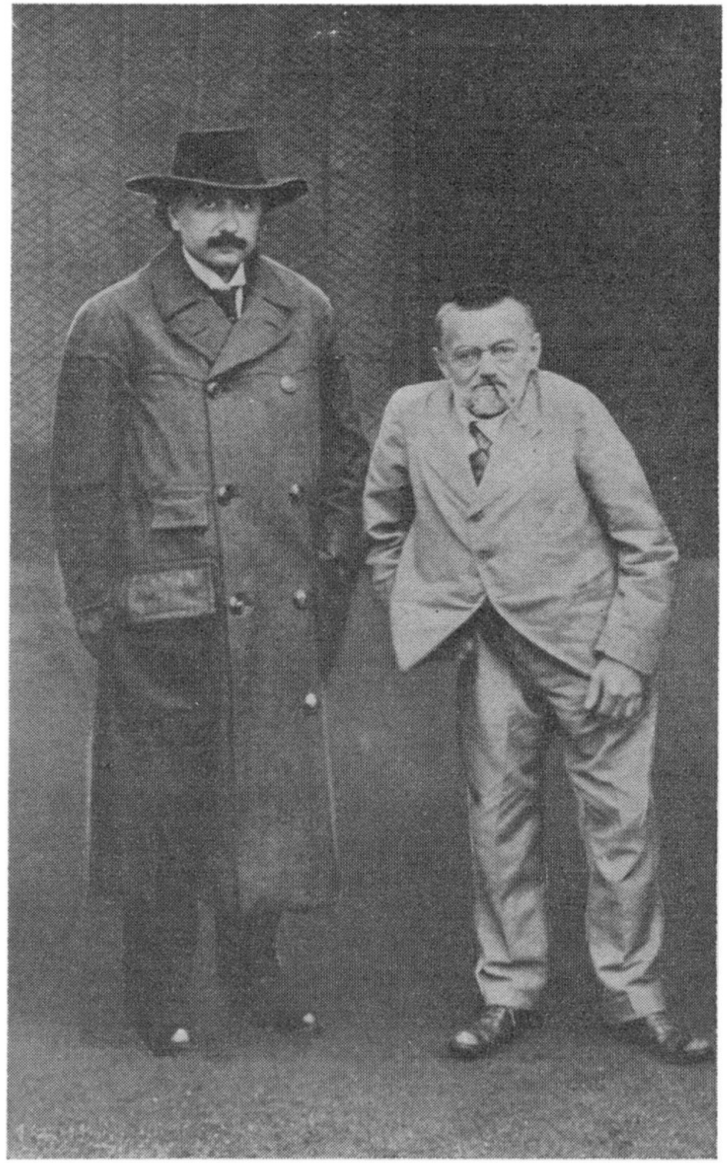
Eksperyment Steinmetz'a: 1892



$$P^2 + Q^2 < S^2$$

?





Einstein and Steinmetz.

$$P^2 + Q^2 < S^2$$

?

$$S = U\,I, \qquad \Delta W = r\,I^2\,\Delta t$$

Za różnicą między S a P kryje się nadmienny prąd przesyłowy i straty energii
Moc pozorna S określa też moc urządzeń przesyłowych

Aby wyjaśnić tę nierówność,
trzeba zrozumieć zjawiska energetyczne w obwodach elektrycznych

Wyjaśnienie zjawisk energetycznychjest celem poznawczym teorii moicy

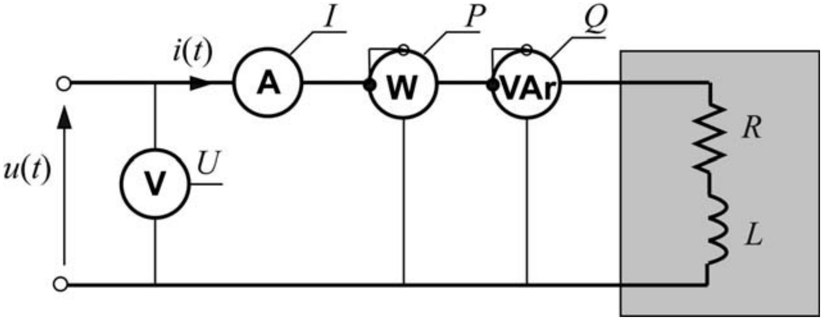
Aby zmniejszyć moc urządzeń przesyłowych, odbiornik trzeba odbiornik kompensować.
Jak?

Opracowanie metod kompensacji jest celem praktycznym teorii mocy

Badania nad teorią mocy zostały zredukowane do zagadnienia znalezienia równania mocy odbiornika liniowego

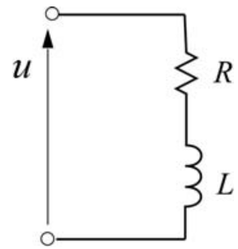
zasilanego napięciem niesinusoidalnym

$$u = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n)$$



Od raportu Steinmetz'a z 1892r do 1984,
po 92 latach rozwoju teorii mocy
wprowadzono do elektrotechniki pięć różnych równań mocy i pięć różnych definicji mocy biernej
dla tak prostego układu jak szeregowy odbiornik RL

nie będąc w stanie zaprojektować kompensatora poprawiającego współczynnik mocy



1927: Budeanu: $S^2 = P^2 + Q_B^2 + D^2$ $\qquad Q_B \overset{\text{df}}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$

1931: Fryze: $S^2 = P^2 + Q_F^2$ $\qquad Q_F \overset{\text{df}}{=} ||u||\,||i_{rF}||$

1971: Shepherd: $S^2 = S_R^2 + Q_S^2$ $\qquad Q_S \overset{\text{df}}{=} ||u||\,||i_{rS}||$

1975: Kusters: $S^2 = P^2 + Q_K^2 + Q_r^2$ $\qquad Q_K \overset{\text{df}}{=} ||u||\,||i_{rC}||$

1979: Depenbrock: $S^2 = P^2 + Q_1^2 + V^2 + N^2$

Zagadnienie zostało dopiero rozwiązane, wraz z kompensacją, w 1984r.

Czarnecki: $S^2 = P^2 + Q^2 + D_s^2$

Steinmetz: 1892

1927

C.I. Budeanu, Professor of Bucharest University, Romania, introduced

definition of the reactive power

$$Q = Q_B \stackrel{\mathrm{df}}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$$

$$P^2 + Q_B^2 \leq S^2$$

Budeanu concluded that there is also **other power associated with the waveform distortion,** and introduced a new power quantity, called

Distortion Power

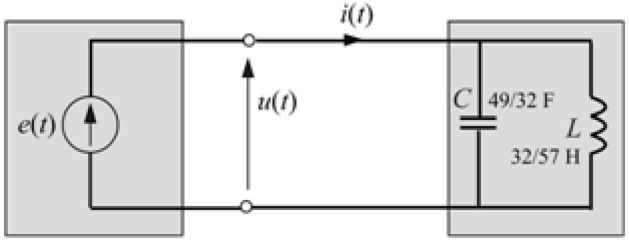
$$D \stackrel{\mathrm{df}}{=} \sqrt{S^2 - (P^2 + Q_B^2)}$$

Budeanu's Power Equation has the form:

$$S^2 = P^2 + Q_B^2 + D^2$$

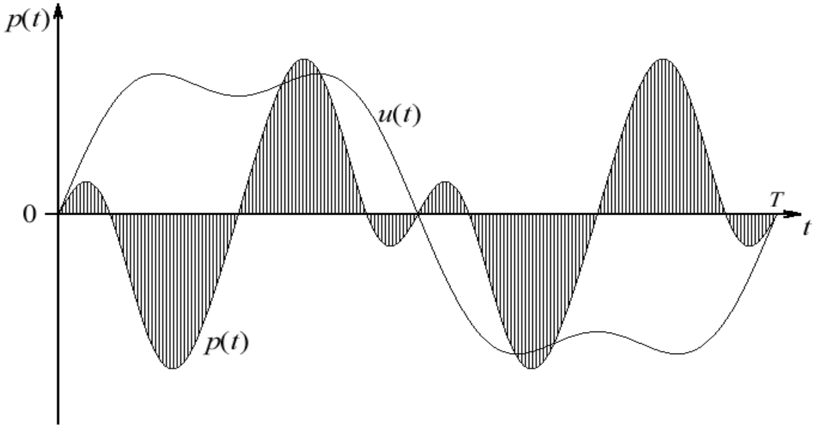
Why Budeanu definition of reactive power Q is wrong?

$$u(t) = \sqrt{2}(100\sin\omega_1 t + 25\sin 3\omega_1 t)\ \mathrm{V}$$



$$i(t) = \sqrt{2}\,[25\sin(\omega_1 t - 90^0) + 100\sin(3\omega_1 t + 90^0)]\ \mathrm{A}$$

$$Q = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n = 100\times 25\times 1 + 00\times 25\times(-1) = 0$$



There are energy oscillations in spite of zero Budeanu's reactive power Q

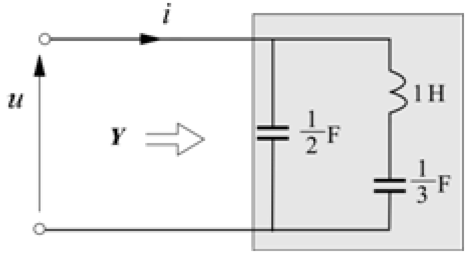
Why Budeanu's definition of Distortion power *D* is wrong?

$$D \overset{\text{df}}{=} \sqrt{S^2 - P^2 - Q^2} = \sqrt{\frac{1}{2}\sum_{r\in N}\sum_{s\in N} U_r^2 U_s^2 \, |\boldsymbol{Y}_r - \boldsymbol{Y}_s|^2}$$

$D = 0$ if for each r, s:

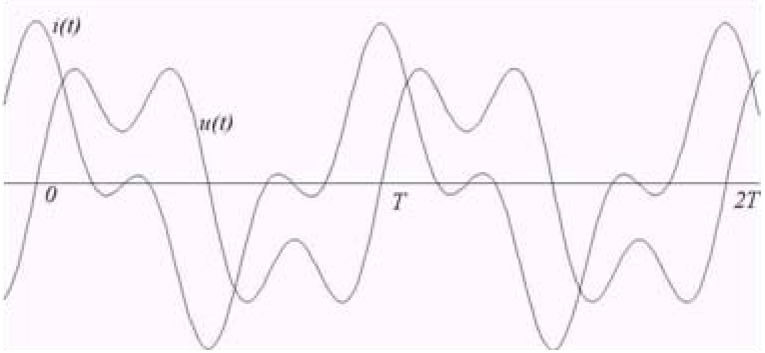
$$\boldsymbol{Y}_r = \boldsymbol{Y}_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$u(t) = \sqrt{2}(100\sin\omega_1 t + 50\sin 3\omega_1 t)\ \text{V}$$



$$\boldsymbol{Y}_1 = \boldsymbol{Y}_3 = 1\, e^{j\frac{\pi}{2}}\ \text{S}$$

$$i(t) = \sqrt{2}\,[100\,\sin(\omega_1 t + \frac{\pi}{2}) + 50\,\sin(3\,\omega_1 t + \frac{\pi}{2})]\ \ \text{A}$$



The load current is distorted in spite of zero distortion power , *D*

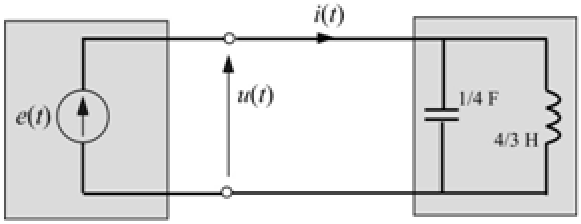
The load current is not distorted, meaning

$$i(t) = a\, u(t-\tau)$$

$$\text{if} \qquad \boldsymbol{I}_n = a \boldsymbol{U}_n e^{-jn\tau} = \boldsymbol{Y}_n \boldsymbol{U}_n,$$

$$\boldsymbol{Y}_n = a\, e^{-jn\tau} \;......................................(2)$$

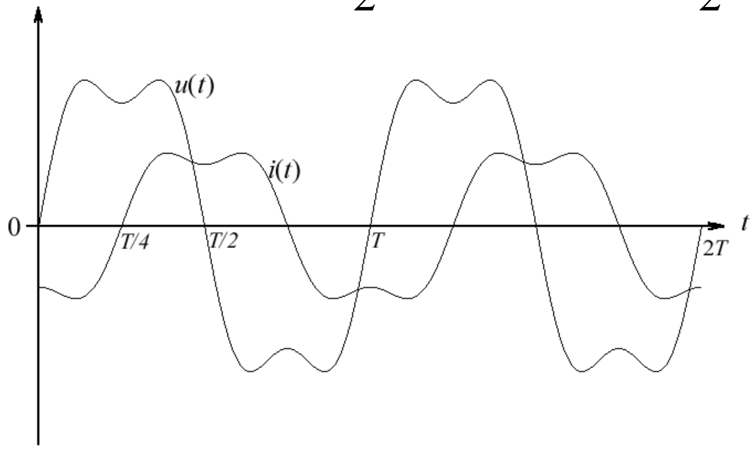
$$u(t) = \sqrt{2}(100 \sin \omega_1 t + 30 \sin 3\omega_1 t)\ \text{V}$$



$$\boldsymbol{Y}_1 = j\frac{1}{4} - j\frac{3}{4} = \frac{1}{2} e^{-j\frac{\pi}{2}}\ \text{S}$$

$$\boldsymbol{Y}_3 = j\frac{3}{4} - j\frac{1}{4} = \frac{1}{2} e^{j\frac{\pi}{2}} = \frac{1}{2} e^{-j3\frac{\pi}{2}}\ \text{S}$$

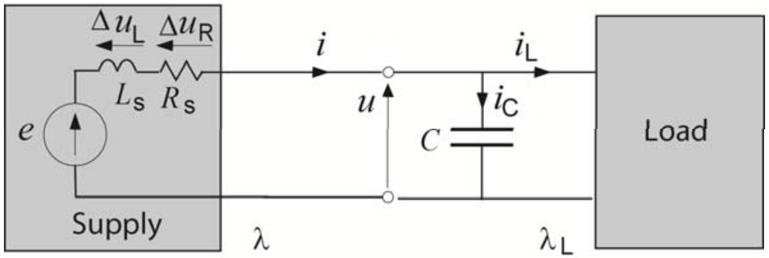
$$i(t) = \sqrt{2}[50 \sin(\omega_1 t - \frac{\pi}{2}) + 15 \sin(3\omega_1 t - \frac{3\pi}{2}) = \sqrt{2}[50 \sin \omega_1 (t - \tfrac{T}{4}) + 15 \sin 3\omega_1 (t - \tfrac{T}{4}) = \frac{1}{2} u(t - \tfrac{T}{4}),$$



$$D = U_1 U_3 |\boldsymbol{Y}_1 - \boldsymbol{Y}_3| = 100 \cdot 25 \cdot |\frac{1}{2} e^{-j\frac{\pi}{2}} - \frac{1}{2} e^{j3\frac{\pi}{2}}| = 2.5\ \text{kVA}$$

The load current is not distorted in spite of non zero distortion power , *D*

Kompensacja przy sinusoidalnym napięciu zasilania



$$C = \frac{Q}{\omega_1 U^2} \quad \rightarrow \quad \lambda = \frac{P}{S} = 1$$

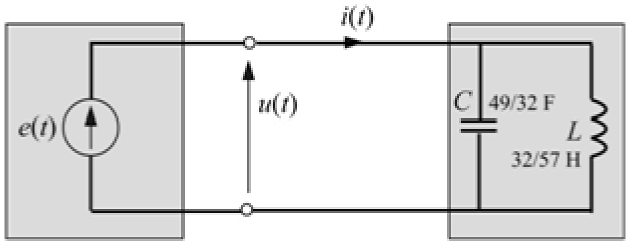
$$u = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

$$\boldsymbol{C = ?}$$

Power factor improvement and Budeanu's reactive power

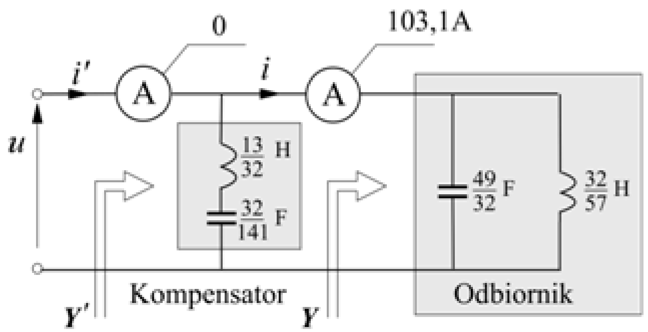
$$\| i \| = \sqrt{\sum_{n=0}^{N} \| i_n \|^2} = \sqrt{\sum_{n=0}^{N} (\frac{P_n}{U_n})^2 + \sum_{n=1}^{N} (\frac{Q_n}{U_n})^2},$$ but in Budeanu Theory: $Q = \sum_{n=1}^{N} Q_n$

$$u(t) = \sqrt{2}(100 \sin\omega_1 t + 25 \sin 3\omega_1 t)\,\mathrm{V}$$



$$i(t) = \sqrt{2}\,[25 \sin(\omega_1 t - 90^0) + 100 \sin(3\omega_1 t + 90^0)]\,\mathrm{A}$$

$$Q = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n = 100 \times 25 \times 1 + 00 \times 25 \times (\text{-}1) = 0$$



Budeanu's reactive power is useless for compensator design

1927: Budeanu:

$$Q = Q_{\mathrm{B}} \overset{\mathrm{df}}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$$

$$D \overset{\mathrm{df}}{=} \sqrt{S^2 - (P^2 + Q_{\mathrm{B}}^2)}$$

$$S^2 = P^2 + Q_{\mathrm{B}}^2 + D^2$$

1987

L.S. Czarnecki: **What is Wrong With the Budeanu's Concept of Reactive and Distortion Powers and Why it Should be Abandoned**,

IEEE Trans. on Instrumentation and Measurements

1931

S. Fryze, Professor of Lwow University, Poland, defined the reactive power in a time-domain, based on

the load current orthogonal decomposition into active and reactive currents

$$i = i_{\mathrm{a}} + i_{\mathrm{rF}}$$

$$i_{\mathrm{a}}(t) \overset{\mathrm{df}}{=} \frac{P}{||u||^2} u(t) \overset{\mathrm{df}}{=} G_{\mathrm{e}}\, u(t), \qquad i_{\mathrm{rF}}(t) \overset{\mathrm{df}}{=} (t) - i_{\mathrm{a}}(t)$$

$$\frac{1}{T}\int_0^T i_{\mathrm{a}}(t)\, i_{\mathrm{rF}}(t)\, dt = (i_{\mathrm{a}}, i_{\mathrm{rF}}) = 0$$

$$||i||^2 = ||i_{\mathrm{a}}||^2 + ||i_{\mathrm{rF}}||^2$$

Fryze's Power Equation: $S^2 = P^2 + Q_{\mathrm{F}}^2$

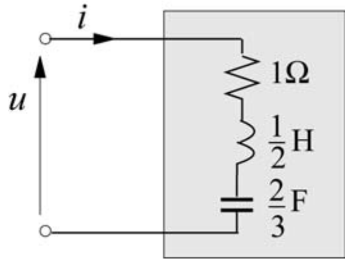
Fryze’s definition of reactive power: $Q_{\mathrm{F}} \overset{\mathrm{df}}{=} ||u||\,||i_{\mathrm{rF}}||$

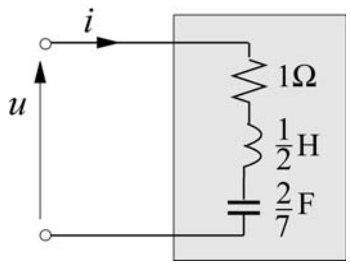
1997

L.S. Czarnecki: Budeanu and Fryze: Two frameworks for interpreting power properties of circuits with nonsinusoidal voltages and currents, *Archiv fur Elektrotechnik*

Question: Does the Fryze's Power Theory provide fundamentals for the power factor improvement?

$$u(t) = 100\sqrt{2}\,(\sin\omega_1 t + \sin 3\omega_1 t)\ \text{V}$$



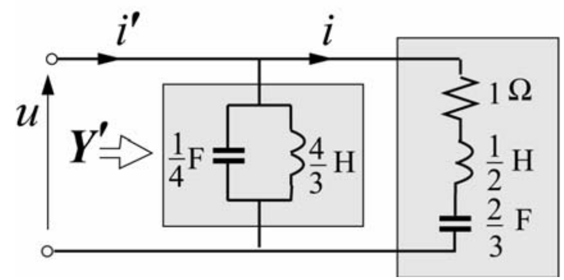


$P = 10$ kW

$Q_F = 10$ kVAr

$S = 14.1$ kVA

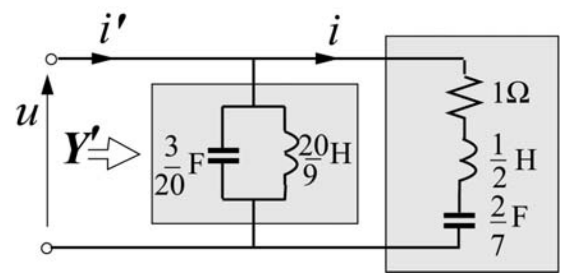
$\lambda = 0.71$



$Q_F = 0$

$S = 10$ kVA

$\lambda = 1$



$Q_F = 8$ kVAr

$S = 12.7$ kVA

$\lambda = 0.78$

These loads cannot be distinguished with respect to Fryze's powers.

They differ as to the possibility of their compensation

Fryze's Power Theory
does not enable us to draw conclusions
as to the possibility of the load compensation with a reactive compensator

Opinion: Fryze's theory provides fundamentals for switching compensator control

$$i = i_a + i_{rF}$$

i_a - active current is useful component

i_{rF} - reactive current is useless

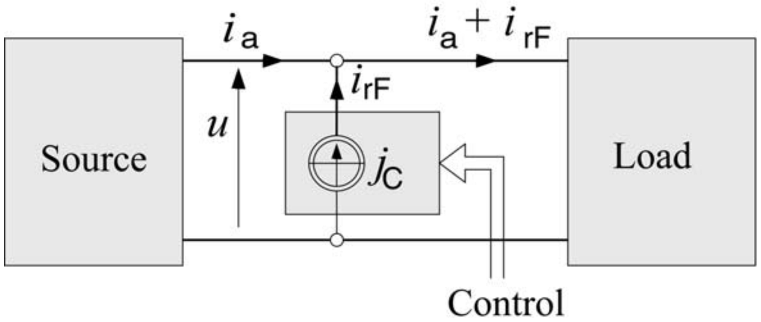
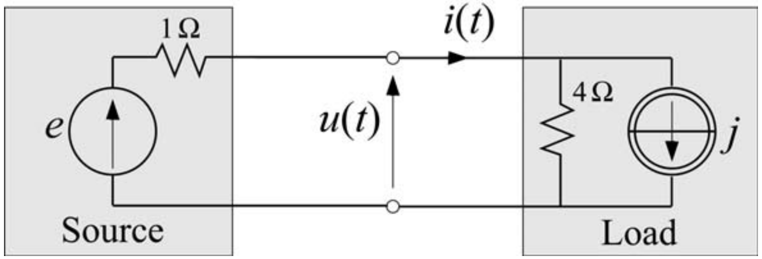


Illustration:

$$e = 100\sqrt{2}\,\sin\omega_1 t\ \text{V}$$



$$j = 50\sqrt{2}\,\sin 3\omega_1 t\ \text{A}$$

$$i = \sqrt{2}(20\,\sin\omega_1 t + 40\,\sin 3\omega_1 t)\ \text{A}$$

$$u = \sqrt{2}(80\,\sin\omega_1 t - 40\,\sin 3\omega_1 t)\ \text{V}$$

$$P = 1600 - 1600 = 0 \qquad i_{\mathrm{a}}(t) = \frac{P}{\|u\|^2}u(t) = 0$$

According to Fryze's Power Theory,

total compensation requires that the current i_{rF} is reduced to zero

This is a wrong conclusion

Only the 3rd order current harmonic should be compensated

1971

Shepherd & Zakikhani, England, developed power theory in the frequency-domain:

$$u = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n) \qquad i = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \alpha_n - \varphi_n),$$

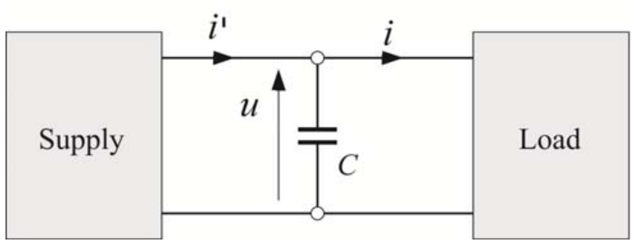
$$i = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos\varphi_n \cos(n\omega_1 t + \alpha_n) + \sqrt{2}\sum_{n=1}^{\infty} I_n \sin\varphi_n \sin(n\omega_1 t + \alpha_n) = i_R + i_r$$

$$\| i \|^2 = \| i_R \|^2 + \| i_{rS} \|^2,$$

$$\| i_R \| = \sqrt{\sum_{n=0}^{\infty} I_n^2 \cos^2\varphi_n}, \qquad \| i_{rS} \| = \sqrt{\sum_{n=1}^{\infty} I_n^2 \sin^2\varphi_n}$$

$$S^2 = S_R^2 + Q_S^2$$

S&H power theory has provided the first solution of the compensation problem:



$$C = C_{opt} = \frac{\sum_{n=1}^{\infty} n U_n I_n \sin\varphi_n}{\omega_1 \sum_{n=1}^{\infty} n^2 U_n^2} = \frac{\sum_{n=1}^{\infty} n Q_n}{\omega_1 \sum_{n=1}^{\infty} n^2 U_n^2}$$

1971

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$$u = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n) \qquad i = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \alpha_n - \varphi_n),$$

$$i = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos\varphi_n \cos(n\omega_1 t + \alpha_n) + \sqrt{2}\sum_{n=1}^{\infty} I_n \sin\varphi_n \sin(n\omega_1 t + \alpha_n) = i_R + i_r$$

$$\| i \|^2 = \| i_R \|^2 + \| i_{rS} \|^2,$$

$$\| i_R \| = \sqrt{\sum_{n=0}^{\infty} I_n^2 \cos^2\varphi_n}, \qquad \| i_{rS} \| = \sqrt{\sum_{n=1}^{\infty} I_n^2 \sin^2\varphi_n}$$

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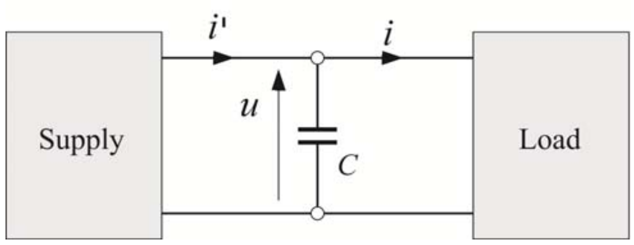
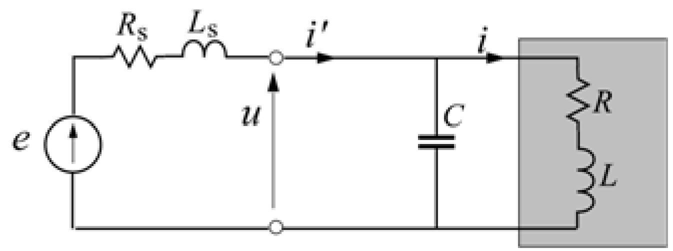
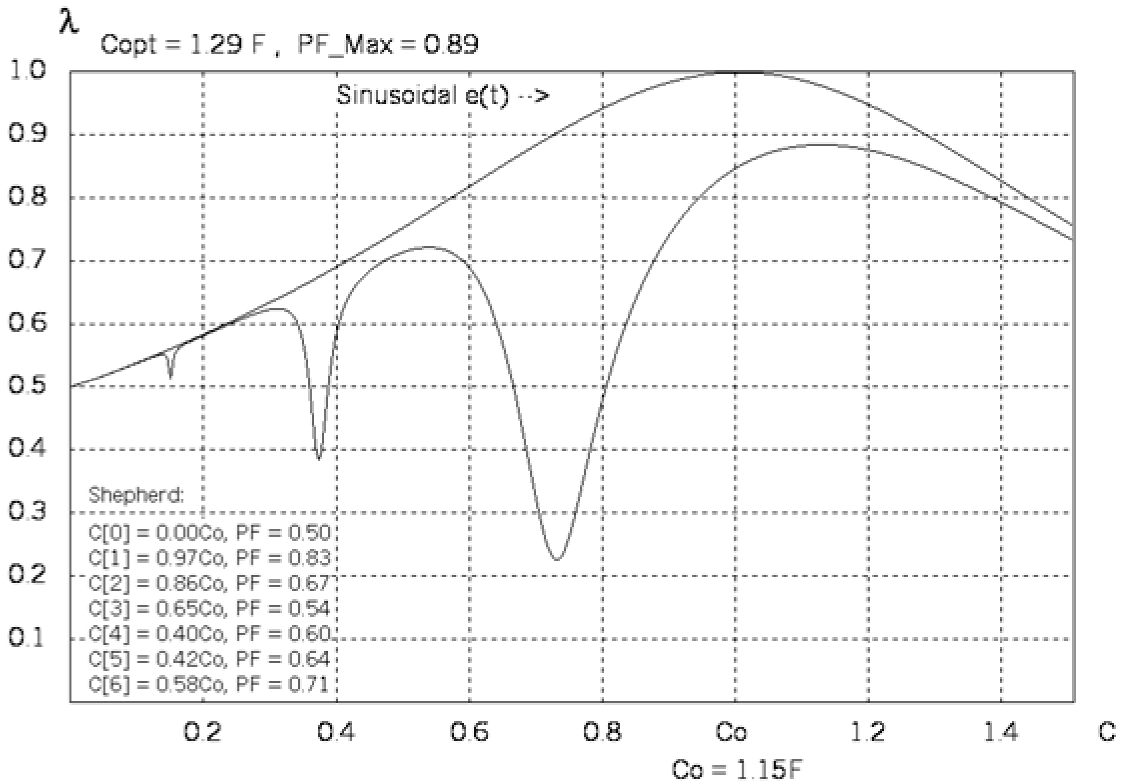


$$C = C_{opt} = \frac{\sum_{n=1}^{\infty} n U_n I_n \sin\varphi_n}{\omega_1 \sum_{n=1}^{\infty} n^2 U_n^2} = \frac{\sum_{n=1}^{\infty} n Q_n}{\omega_1 \sum_{n=1}^{\infty} n^2 U_n^2}$$

$$E_5 = 3.0\ \%\ E_1\,,\quad E_7 = 1.5\ \%\ E_1\,,\quad E_{11} = 0.5\ \%\ E_1$$

$$S_{sc} = 28.6 \times P$$



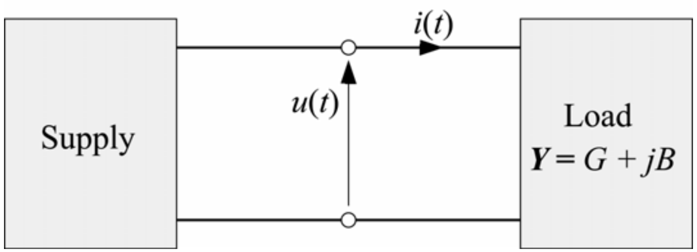


In situations common in distribution systems, the K&M power theory does not provide optimal capacitance of a compensator.
The same conclusion applies to the S&Z power theory

Kusters and Moore (NRC, Canada) solved the same problem (in 1980) in the time-domain

$$u = \sqrt{2}\sum_{n=1}^{\infty} U_n \cos n\omega_1 t, \qquad \dot{u} = \frac{du}{dt}$$

$$i = \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t - \varphi_n) = i_a + i_{qC} + i_{qCr}$$



defined as

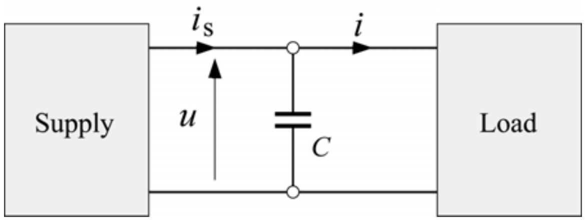
$$i_a = \frac{P}{\|u\|^2} u = G_e\, u, \qquad i_{qC} = \frac{(\dot{u}, i)}{\|\dot{u}\|^2}\dot{u} = C_e\, \dot{u}, \qquad i_{qCr} = i - (i_a + i_{qC})$$

Currents i_a, i_{qC} and i_{qCr} are orthogonal

$$\|i\|^2 = \|i_a\|^2 + \|i_{qC}\|^2 + \|i_{qCr}\|^2 \qquad \times \|u\|^2$$

$$S^2 = P^2 + Q_C^2 + Q_r^2$$

Decomposition suggested by Kusters & Moore's solved the problem of a capacitive compensation in a time-domain



Current i_{qCr} is not affected by a shunt capacitor

$$||i_s|| = ||i_s||_{min}, \quad \text{if} \quad (i_s)_{qC} = 0,$$

which requires that

$$C = C_{opt} = -\frac{(\dot{u}, i)}{||\dot{u}||^2}$$

Results obtained by Shepherd & Zakikhani
and Kusters & Moore
with respect to the optimal capacitance are equivalent

$$C_{\mathrm{opt}} = -\frac{(\dot{u}, i)}{\|\dot{u}\|^2} = \frac{\sum_{n=1}^{\infty} n U_n I_n \sin\varphi_n}{\omega_1 \sum_{n=1}^{\infty} n^2 U_n^2}$$

and, unfortunately, obtained under the same condition,
namely,
that the load voltage does not depend
on the capacitance *C*

It was demonstrated in the paper
L.S. Czarnecki, "Additional discussion to "Reactive power under nonsinusoidal conditions“, *IEEE Trans. on Power and Systems,* Vol. PAS-102, No. 4, pp. 1023-1024, April 1983.

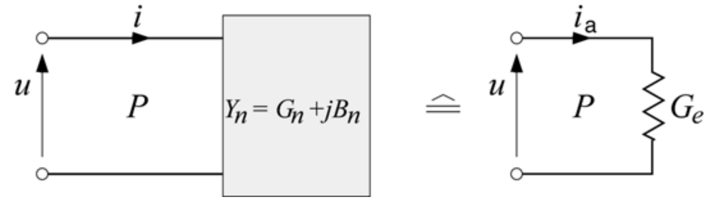
1984,

L.S. Czarnecki: Considerations on the Reactive Power Under Nonsinusoidal Conditions

IEEE Transactions on Instrumentation and Measurements,

$$u = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

$$i = G_0 U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{Y}_n \boldsymbol{U}_n\, e^{jn\omega_1 t}$$



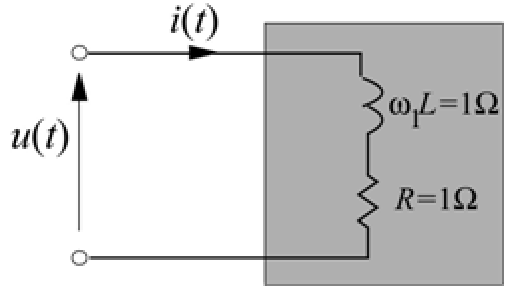
$$i = i_a + i_s + i_r$$

$$i_a(t) \overset{\mathrm{df}}{=} G_e\, u(t), \qquad G_e \overset{\mathrm{df}}{=} \frac{P}{\|u\|^2}$$ Active current

$$i_s(t) \overset{\mathrm{df}}{=} (G_0 - G_e)U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} (G_n - G_e)\boldsymbol{U}_n\, e^{jn\omega_1 t}$$ **Scattered current**

$$i_r(t) \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t}$$ Reactive current

This decomposition has revealed a new power phenomenon, namely, the existence of ***the scattered current***, $\boldsymbol{i}_s$, that occurs when the load conductance, G_n, changes with harmonic order, n.



$$G_n = \mathrm{Re}\{\boldsymbol{Y}_n\} = \mathrm{Re}\frac{1}{R + jn\omega_1 L} = \frac{R}{R^2 + (n\omega_1 L)^2}$$

$$G_0 = 1\ \mathrm{S}, \quad G_1 = 0.5\ \mathrm{S}, \quad G_2 = 0.2\ \mathrm{S}, \quad G_3 = 0.1\ \mathrm{S}, \quad G_4 = 0.06\ \mathrm{S}$$

$$i = i_a + i_s + i_r$$

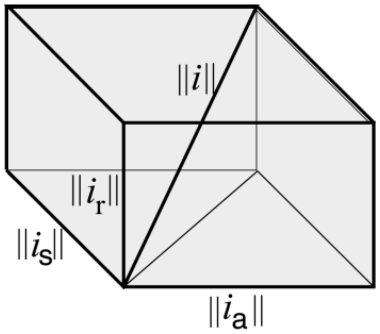
Currents i_a, i_s and i_r are orthogonal

$$||i||^2 = ||i_a||^2 + ||i_s||^2 + ||i_r||^2$$

$$||i_a|| = G_e ||u||$$

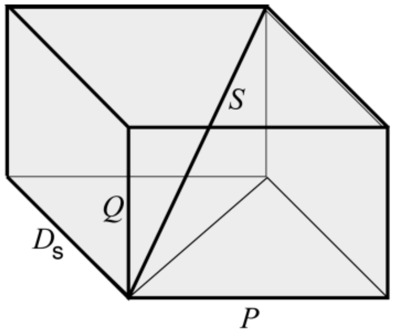
$$||i_s|| = \sqrt{\sum_{n=0}^{\infty} (G_n - G_e)^2 U_n^2}$$

$$||i_r|| = \sqrt{\sum_{n=1}^{\infty} B_n^2 U_n^2}$$



Power equation in the CPC power theory

$$S^2 = P^2 + D_s^2 + Q^2$$



$$P = ||u|| \times ||i_a|| = G_e ||u||^2$$

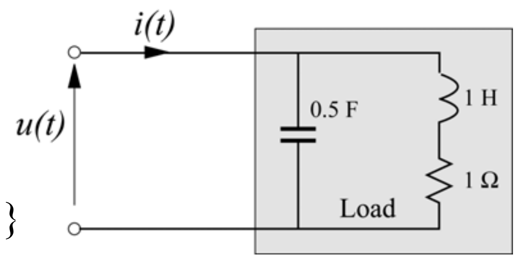
$$Q \overset{\text{df}}{=} ||u|| \times ||i_r||$$

$$D_s \overset{\text{df}}{=} ||u|| \times ||i_s||$$

$$u = 50 + \sqrt{2}\,\mathrm{Re}\{100 e^{j\omega_1 t} + 20 e^{j5\omega_1 t}\}$$

$$\|u\| = 113.58 \text{ V}$$

$$i(t) = 50 + \sqrt{2}\,\mathrm{Re}\{50 e^{j\omega_1 t} + 46.2 e^{j89^0} e^{j5\omega_1 t}\}$$



$$G_0 = 1 \text{ S}, \quad \boldsymbol{Y}_1 = 0.5 \text{ S}, \quad \boldsymbol{Y}_5 = 0.04 - j\,2.31 \text{ S},$$

$$G_e = \frac{P}{\|u\|^2} = 0.5826 \text{ S}$$

$$i_a = G_e u = 0.5826 \times [50 + \sqrt{2}\,\mathrm{Re}\{100 e^{j\omega_1 t} + 20 e^{j5\omega_1 t}\}] = 29.1 + \sqrt{2}\,\mathrm{Re}\{58.3 e^{j\omega_1 t} + 11.6 e^{j5\omega_1 t}\} \text{ A}$$

$$i_s = (G_0 - G_e)U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty}(G_n - G_e)\boldsymbol{U}_n e^{jn\omega_1 t} = 20.9 + \sqrt{2}\,\mathrm{Re}\{-8.3 e^{j\omega_1 t} - 10.8 e^{j5\omega_1 t}\} \text{ A}$$

$$i_r = \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n \boldsymbol{U}_n e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\{j2.31 \times 20 e^{j5\omega_1 t}\} = \sqrt{2}\,\mathrm{Re}\{j\,46.2 e^{j5\omega_1 t}\} \text{ A}$$

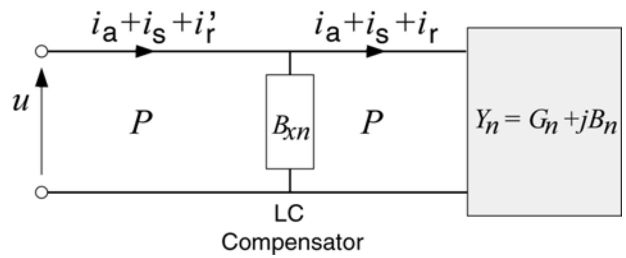
$$\|i_a\| = G_e \|u\| = 66.17 \text{ A}$$

$$\|i_s\| = \sqrt{\sum_{n=0,1,5} (G_n - G_e)^2 U_n^2} = 24.93 \text{ A}$$

$$\|i_r\| = \sqrt{\sum_{n=1,5} B_n^2 U_n^2} = 46.2 \text{ A}$$

$D_s = 113.58 \text{ x } 24.93 = 2.83$ kVA, $\quad Q = 113.58 \text{ x } 46.2 = -5.24$ kVAr

Compensation

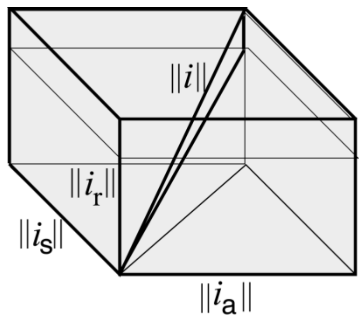


Lossless shunt reactive compensators do not change active power, P, and conductance G_n.

$$G_e = \frac{P}{\|u\|^2} = \text{const.}$$

$$\|i_a\| = G_e \|u\| = \text{const.}$$

$$\|i_s\| = \sqrt{\sum_{n=0}^{\infty} (G_n - G_e)^2 U_n^2} = \text{const.}$$



The RMS value of the reactive current changes to: $$\|i_r'\| = \sqrt{\sum_{n=1}^{\infty} (B_n + B_{xn})^2 U_n^2}$$

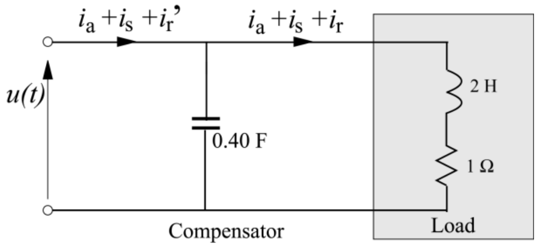
Total compensation of the reactive current:

$\|i_r'\| = 0$, if for each n, such that $U_n \neq 0$, $B_{xn} = -B_n$

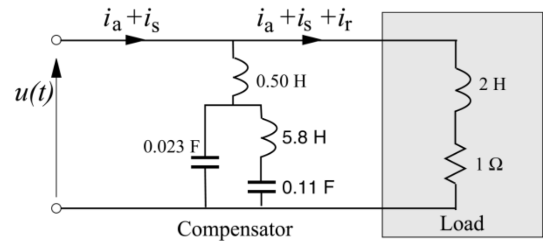
CPC power theory
solves the problem of a shunt reactive compensation of LTI loads

Illustration

$$u(t) \;=\; \sqrt{2}\,\mathrm{Re}\{100e^{j\omega_1 t} + 5e^{j5\omega_1 t}\}\ \mathrm{V} \qquad \omega_1 = 1\ \mathrm{rd/s}$$



$$Y_1 = 0.20 - j0.40\ \mathrm{S}$$
$$Y_5 = 0.01 - j0.10\ \mathrm{S}$$



$Y_1' = 0.20$ S
$Y_5' = 0.01 + j1.9$ S

$Y_1' = 0.20$ S
$Y_5' = 0.01$ S

$$i(t) \;=\; \sqrt{2}\,\mathrm{Re}\{20e^{j\omega_1 t} + 9.5e^{j89^0}e^{j5\omega_1 t}\}\ \mathrm{A}$$

$$i(t) \;=\; \sqrt{2}\,\mathrm{Re}\{20e^{j\omega_1 t} + 0.05\,e^{j5\omega_1 t}\}\ \mathrm{A}$$

$||i_a|| \;=\; 19.98$ A

$||i_s|| \;=\; 0.95$ A

$||i_r'|| \;=\; 9.50$ A

$||i_a|| \;=\; 19.98$ A

$||i_s|| \;=\; 0.95$ A

$||i_r'|| \;=\; 0$

Power factor

$$\lambda \;=\; \frac{P}{S} = \frac{P}{\sqrt{P^2 + D_s^2 + Q^2}} = \frac{||i_a||}{\sqrt{||i_a||^2 + ||i_s||^2 + ||i_r||^2}}$$

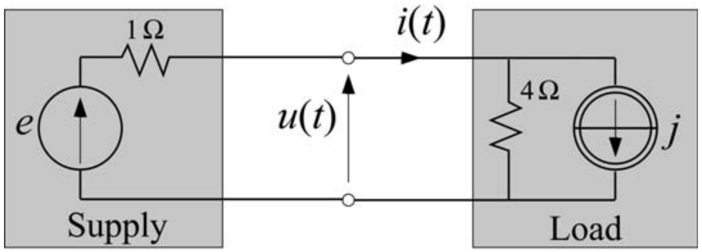
$\lambda \;=\; 0.867$

$\lambda \;=\; 0.999$

Circuits with harmonics generating loads (HGL)

L.S. Czarnecki, T. Swietlicki: Powers in nonsinusoidal networks, their analysis, interpretation and measurement, *IEEE Trans. Instrumentation & Measurement,* Vol. IM-39, No. 2, **1990**

$$e = 100\sqrt{2}\,\sin\omega_1 t\ \text{V}$$



$$j = 50\sqrt{2}\,\sin 3\omega_1 t\ \text{A}$$

$$i = \sqrt{2}(20\sin\omega_1 t + 40\sin 3\omega_1 t)\ \text{A}$$

$$u = \sqrt{2}(80\sin\omega_1 t - 40\sin 3\omega_1 t)\ \text{V}$$

$$P = P_1 + P_3 = 1600 - 1600 = 0$$

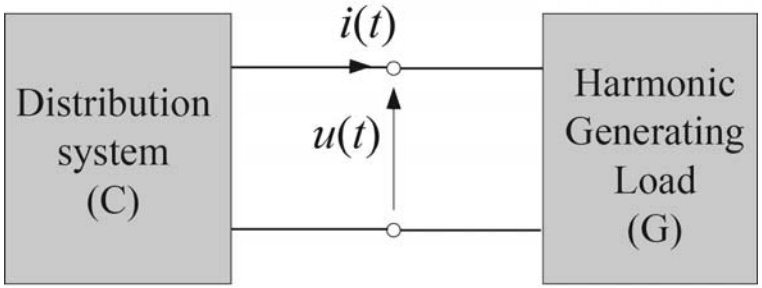
$$Q = 0,$$

$$||i|| = 44.72\ \text{A}, \qquad ||u|| = 89.44\ \text{V}$$

$$S = 4000\ \text{VA}$$

$$S^2 \neq P^2 + D_s^2 + Q^2$$

How to write power equation for such a load?



$$u = \sum_{n \in N} u_n, \quad i = \sum_{n \in N} i_n, \quad P = \sum_{n \in N} P_n$$

Set N of harmonic orders n can be decomposed into two sub-sets, N_D, and N_C, based on the sign of the harmonic active power P_n.

$$P_n = U_n I_n \cos\varphi_n$$

if $|\varphi_n| \le \pi/2$, then $n \in N_C$,

$$\sum_{n \in N_C} u_n \overset{\text{df}}{=} u_C, \quad \sum_{n \in N_C} i_n \overset{\text{df}}{=} i_C, \quad \sum_{n \in N_C} P_n \overset{\text{df}}{=} P_C$$

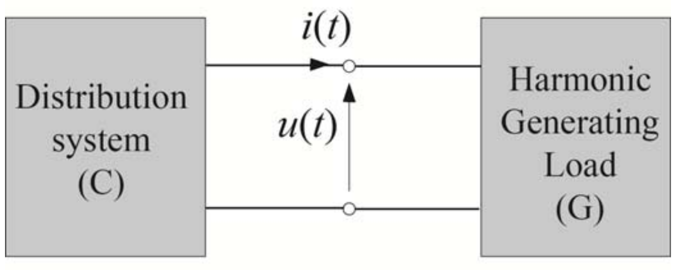
if $|\varphi_n| > \pi/2$, then $n \in N_G$.

$$\sum_{n \in N_G} u_n \overset{\text{df}}{=} -u_G, \quad \sum_{n \in N_G} i_n \overset{\text{df}}{=} i_G, \quad \sum_{n \in N_G} P_n \overset{\text{df}}{=} -P_G$$

$$u = u_C - u_G, \quad i = i_C + i_G, \quad P = P_C - P_G$$

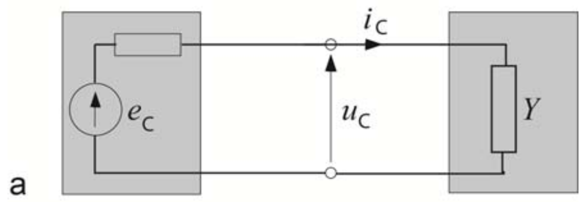
$$||u||^2 = ||u_C||^2 + ||u_G||^2$$

$$||i||^2 = ||i_C||^2 + ||i_G||^2$$



Equivalent circuits of a system with HGL:

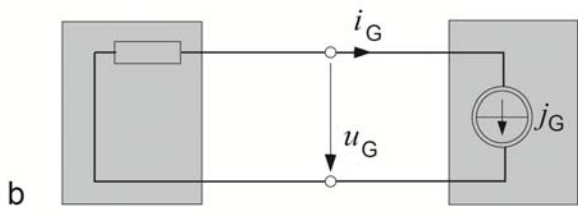
For $n \in N_C$



$$\boldsymbol{Y}_n \stackrel{\text{df}}{=} G_n + jB_n = \frac{\boldsymbol{I}_n}{\boldsymbol{U}_n}$$

$$i_C = i_{aC} + i_{sC} + i_{rC}$$

For $n \in N_G$



$$i = i_{aC} + i_{sC} + i_{rC} + i_{G}$$

$$i_{aC} \stackrel{df}{=} G_{eC} u_{C}, \qquad G_{eC} \stackrel{df}{=} \frac{P_C}{||u_C||^2}$$

Active current

It is not Fryze's active current !!

$$i_{sC} \stackrel{df}{=} \sqrt{2}\,\mathrm{Re} \sum_{n \in N_C} (G_n - G_e) \boldsymbol{U}_n e^{jn\omega_1 t}$$

Scattered current

$$i_{rC} \stackrel{df}{=} \sqrt{2}\,\mathrm{Re} \sum_{n \in N_C} jB_n \boldsymbol{U}_n e^{jn\omega_1 t}$$

Reactive current

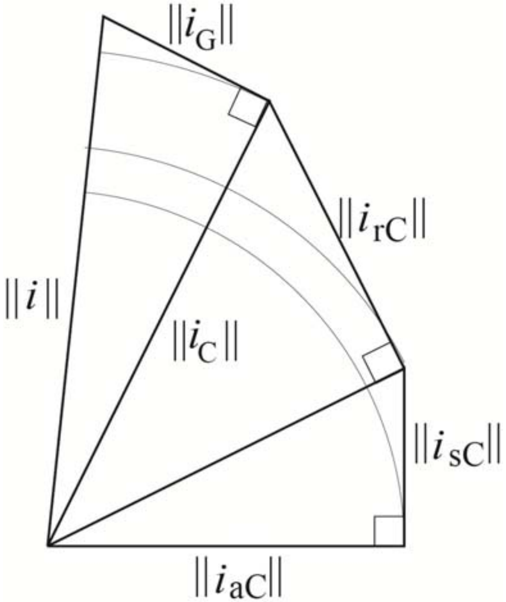
$$i_{G} \stackrel{df}{=} \sum_{n \in N_G} i_n$$

Load generated current

These are CPC of HGL

They are mutually orthogonal

$$||i||^2 = ||i_{aC}||^2 + ||i_{sC}||^2 + ||i_{rC}||^2 + ||i_G||^2$$



Apparent power of Harmonics Generating Loads:

$$S \overset{\text{df}}{=} ||u||\,||i|| = \sqrt{(||u_{\text{C}}||^2 + ||u_{\text{G}}||^2)\,(||i_{\text{C}}||^2 + ||i_{\text{G}}||^2)} = \sqrt{S_{\text{C}}^2 + S_{\text{CG}}^2 + S_{\text{G}}^2}$$

$$S_{\text{C}} \overset{\text{df}}{=} ||u_{\text{C}}||\,||i_{\text{C}}|| = ||u_{\text{C}}||\,\sqrt{||i_{\text{aC}}||^2 + ||i_{\text{sC}}||^2 + ||i_{\text{rC}}||^2} = \sqrt{P_{\text{C}}^2 + D_{\text{s}}^2 + Q^2}$$

$$S_{\text{G}} \overset{\text{df}}{=} ||u_{\text{G}}||\,||i_{\text{G}}||$$

$$S_{\text{CG}} \overset{\text{df}}{=} \sqrt{||u_{\text{C}}||^2||i_{\text{G}}||^2 + ||u_{\text{G}}||^2||i_{\text{C}}||^2}$$

Power equation of HGLs:

$$S^2 = P_{\text{C}}^2 + D_{\text{s}}^2 + Q^2 + S_{\text{G}}^2 + S_{\text{CG}}^2$$

Power factor:

$$\lambda \overset{\text{df}}{=} \frac{P}{S} = \frac{P_{\text{C}} - P_{\text{G}}}{\sqrt{P_{\text{C}}^2 + D_{\text{s}}^2 + Q^2 + S_{\text{G}}^2 + S_{\text{CG}}^2}}$$

Co to jest moc bierna $Q = UI\,sin\varphi$

?

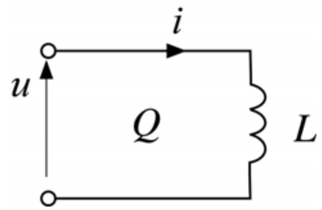
lub inaczej:

Z jakim zjawiskiem fizycznym związana jest moc bierna?

A. Oscylacja energii między źródłem a odbiornikiem?
B. Gromadzenie energii w polach elektromagnetycznych?
C. Przesunięcie fazowe prądu i napięcia?
D. Wytwarzanie pola magnetycznego w silnikach?
E. Jeszcze coś innego? Co?

Question: **Is the reactive power, *Q*, associated with energy storage?**

$$i(t) = \sqrt{2}\, I \sin\omega t$$



Energy stored in magnetic field, *T* :

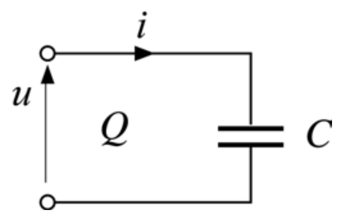
$$T = \frac{1}{2} L i^2(t) = L I^2 \sin^2\omega t = T_{max} \sin^2\omega t$$

Reactive power:

$$Q = U I \sin\frac{\pi}{2} = \omega L I^2 = \omega T_{max}$$

Energy stored in electric field , *V*:

$$u(t) = \sqrt{2}\, U \sin\omega t$$



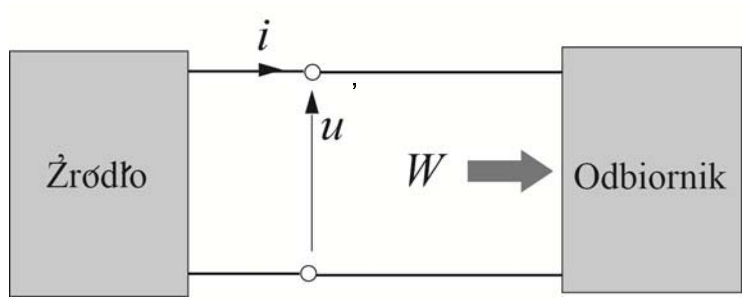
$$V = \frac{1}{2} C u^2(t) = C U^2 \sin^2\omega t = V_{max} \sin^2\omega t$$

Reactive power:

$$Q = U I \sin(-\frac{\pi}{2}) = -\omega C U^2 = -\omega V_{max}$$

$$u(t) = \sqrt{2}\,U \cos\omega_1 t$$

$$i(t) = \sqrt{2}\,I \cos(\omega_1 t - \varphi)$$

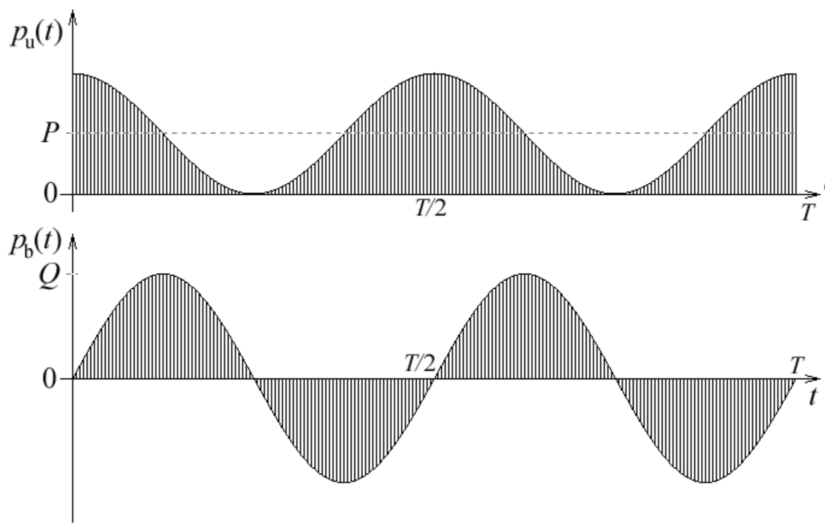


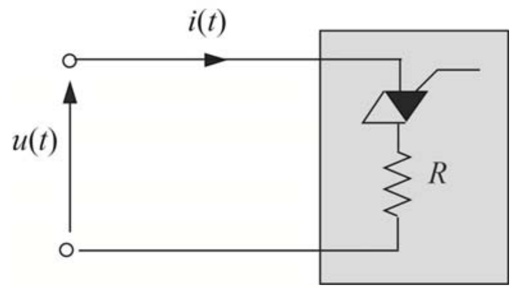
$$p(t) = \frac{dW(t)}{dt} = u(t)\,i(t)$$

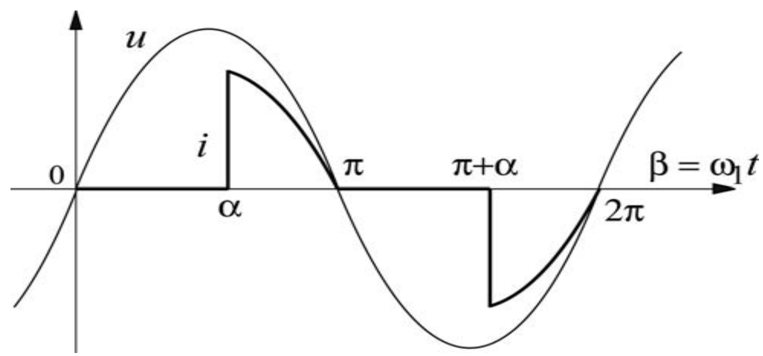
$$p(t) = u(t)\,i(t) = 2\,UI \cos\omega_1 t \;\cos(\omega_1 t - \varphi) \;=\; p_u(t) + p_b(t)$$

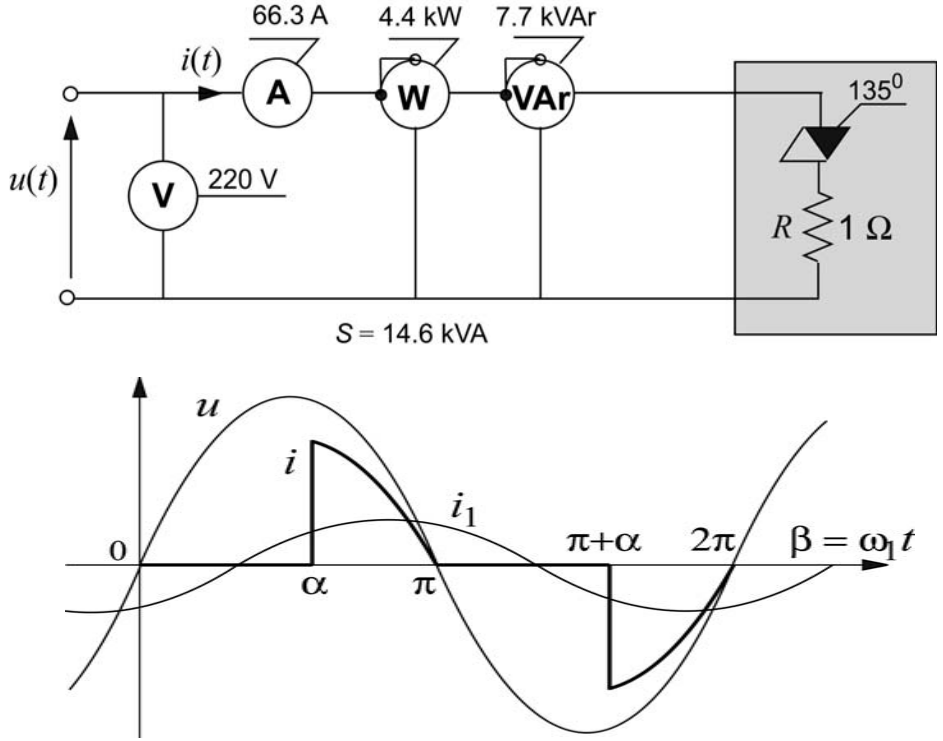
$$p_u(t) \overset{\text{df}}{=} P\,(1 + \cos 2\omega_1 t)$$

$$p_b(t) \overset{\text{df}}{=} Q\,\sin 2\omega_1 t$$

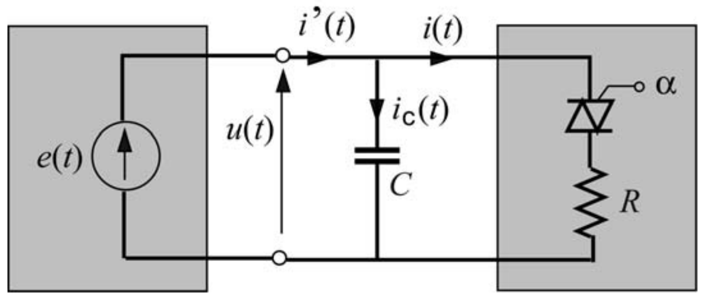


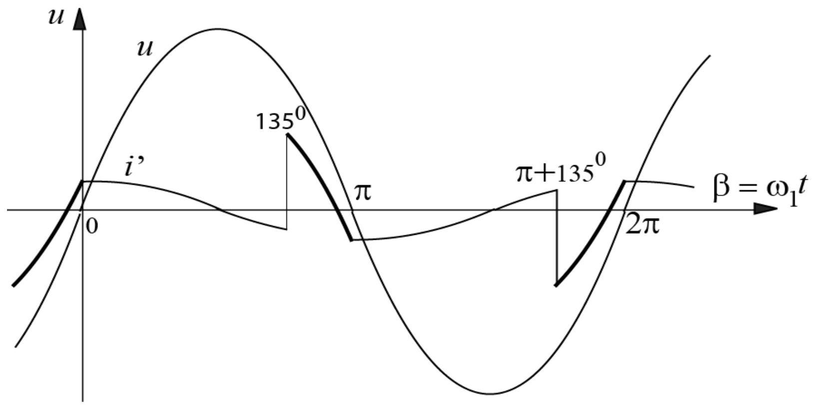

i(t)
u(t)
R


u
i
0
α
π
π+α
2π
β = ω1 t



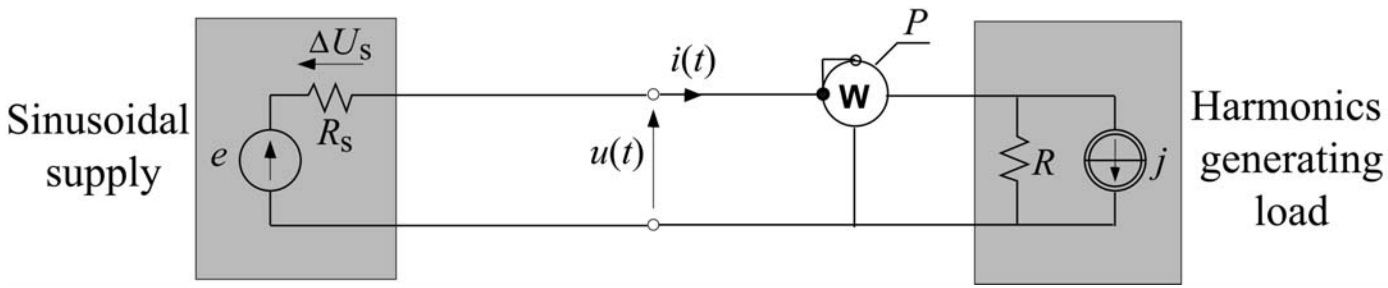
W obwodzie jest niezerowa moc bierna Q, przy braku oscylacji energii i jej gromadzenia w polach elektromagnetycznych odbiornika





Kompensator poprawia współczynnik mocy λ,
jednocześnie wprowadzając oscylacje energii między źródłem a skompensowanym odbiornikiem!!

Czy na pewno moc czynna $\boldsymbol{P}$ jest mocą użyteczną ?



$$u(t) = \sum_{n \in N} u_n = u_1 + u_h \qquad i(t) = \sum_{n \in N} i_n = i_1 + i_h$$

$$P = \frac{1}{T}\int_0^T u(t)\, i(t)\, dt = P_1 + P_2 + P_3 + P_4 + \dots$$

$$P_1 = U_1 I_1 > 0$$

$$P_n = U_n I_n = (-R_s I_n) I_n = -R_s I_n^2 < 0$$

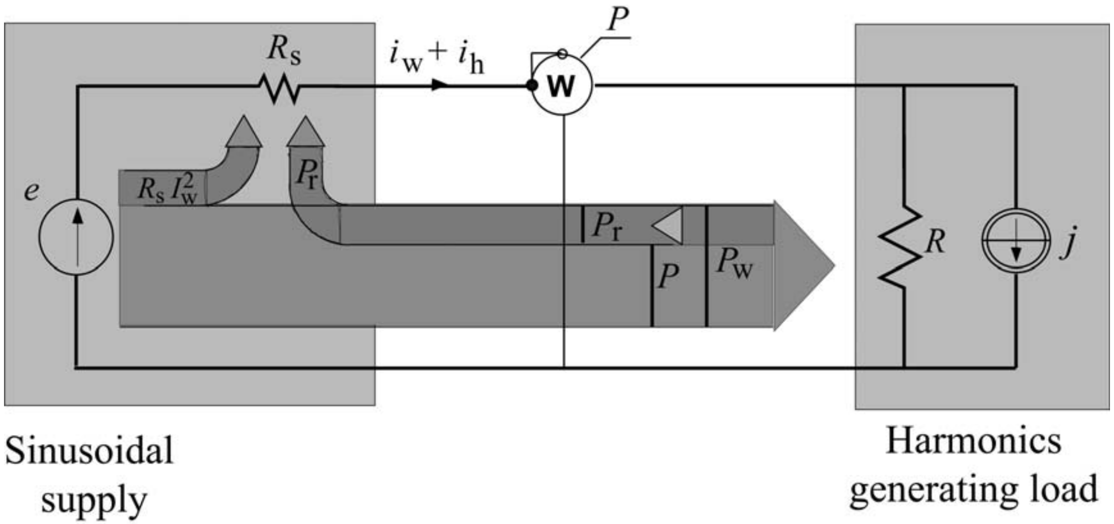
Energia do odbiornika dostarczana jest z mocą P_1.
jest to

robocza moc czynna, $P_1 = P_w$

$$-(P_2 + P_3 + P_4 + \dots) = P_r$$

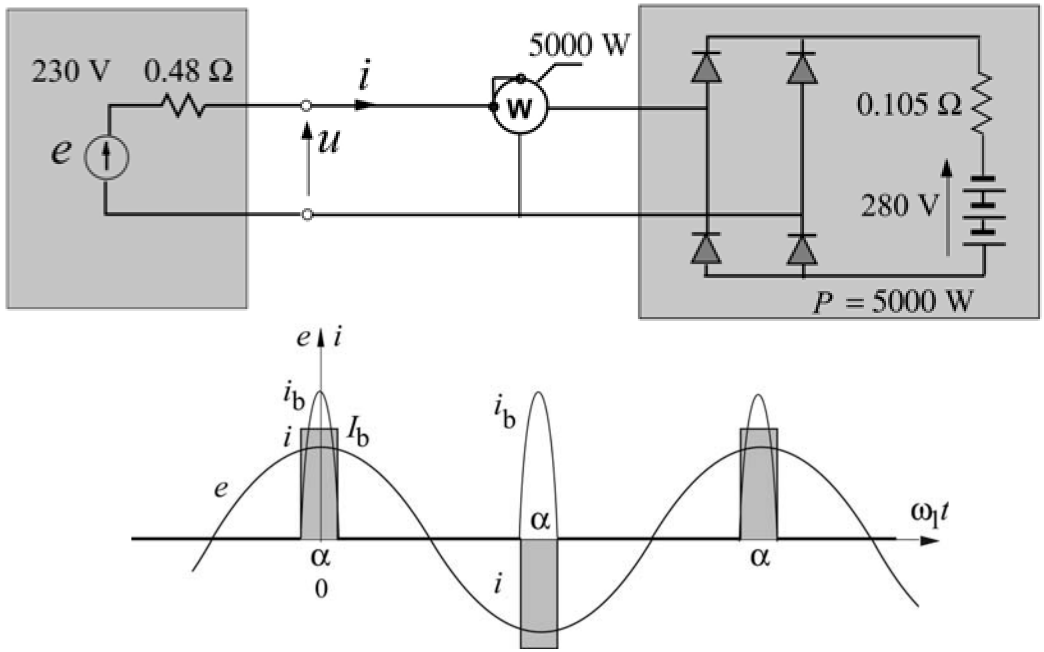
odbita moc czynna

$$P = P_w - P_r$$



Odbiornik o mocy czynnej P generujący harmoniczne musi być zasilany z mocą roboczą P_w.

$$P_w > P$$



Moc czynna P = 5000 W

Robocza moc czynna P_w = 5242 W

Odbita moc czynna P_r = 242 W

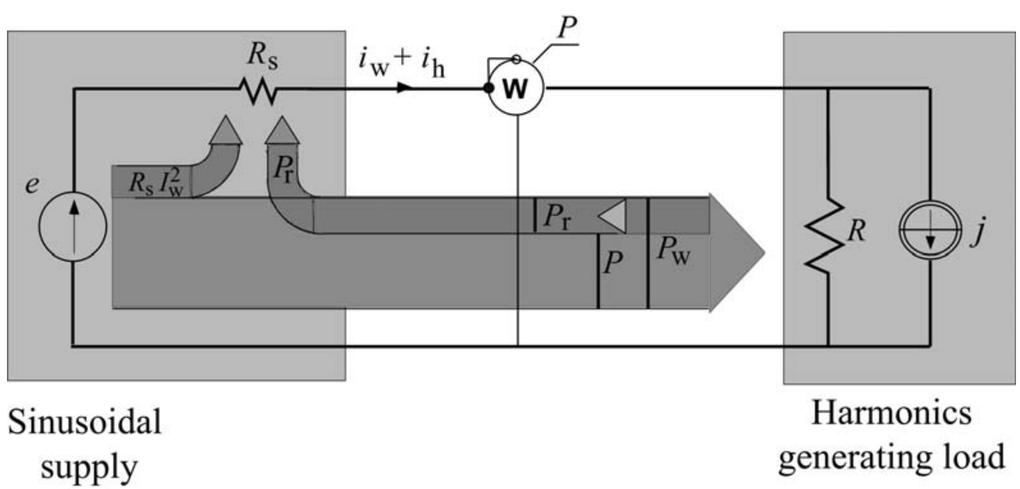
Moc strat w źródle: ΔP_s = 900 W

Harmoniczne generowane w odbiorniku
powodują dodatkowe straty wewnątrz systemu zasilającego.
Odbiorca winien płacić za energię roboczą

$$\int_0^{\tau} P_{\rm w}\, dt = \int_0^{\tau} (P + P_{\rm r})\, dt = W_{\rm w} > W_{\rm a}$$

Prąd potrzebny do przenoszenia energii roboczej $W_{\rm w}$
ma większą moc skuteczną
od prądu przenoszącego energię czynną $W_{\rm a}$

$$\Delta P_{\rm s} = P_{\rm r} + R_{\rm s}\, I_{\rm w}^2$$



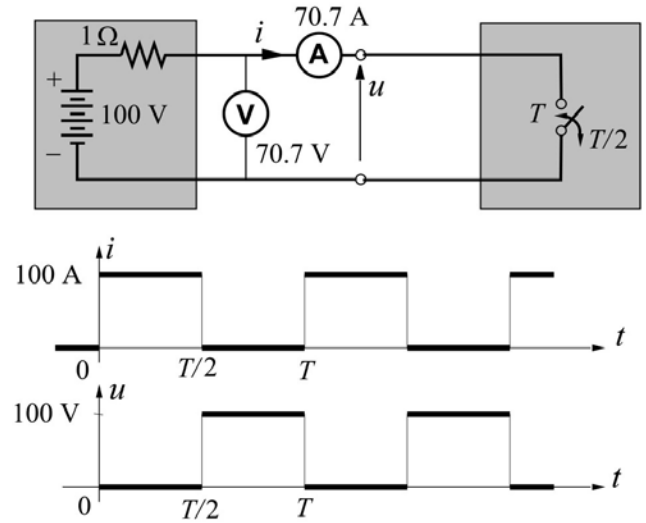
Should the power theory be formulated
in the frequency-domain, as postulated by Budeanu
or
in the time-domain, as postulated by Fryze?

Should the power theory be formulated based on quantities calculated
by averaging over a period of the supply voltage, as postulated by Fryze
or
on instantaneous values, as postulated by Akagi and Nabae?

Teoria Składowych Fizycznych Prądu
(Currents' Physical Component, CPC – based Power Theory)

Została zbudowana w dziedzinie częstotliwościowej (wg. Budeanu)
z uśrednianiem w okresie *T* (wg. Fryzego)

Obwód Fryzego:

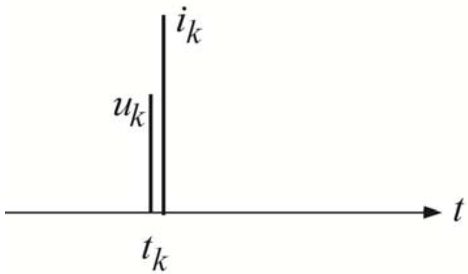


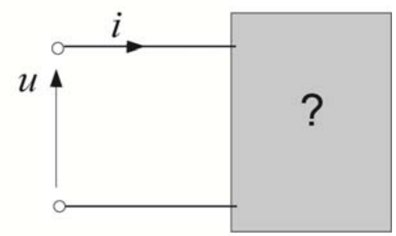
$$u(t) = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n) = \sum_{n=0}^{\infty} u_n$$

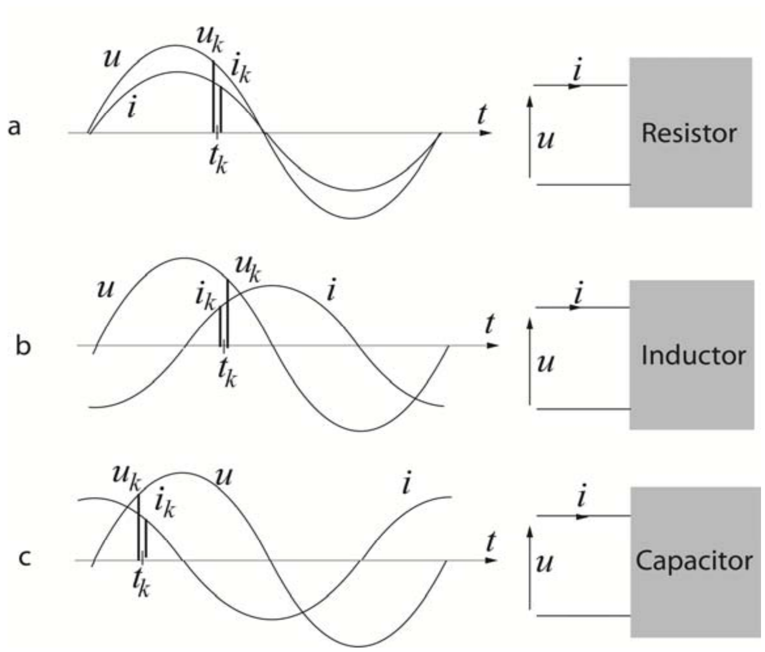
$$i(t) = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \beta_n) = \sum_{n=0}^{\infty} i_n.$$

$$p(t) = \frac{dW}{dt} = u(t)\, i(t) = \sum_{r=0}^{\infty} u_r \sum_{s=0}^{\infty} i_s = \sum_{n=0}^{\infty} S_n \cos(n\omega_1 t + \psi_n)$$

Teorie mocy chwilowej







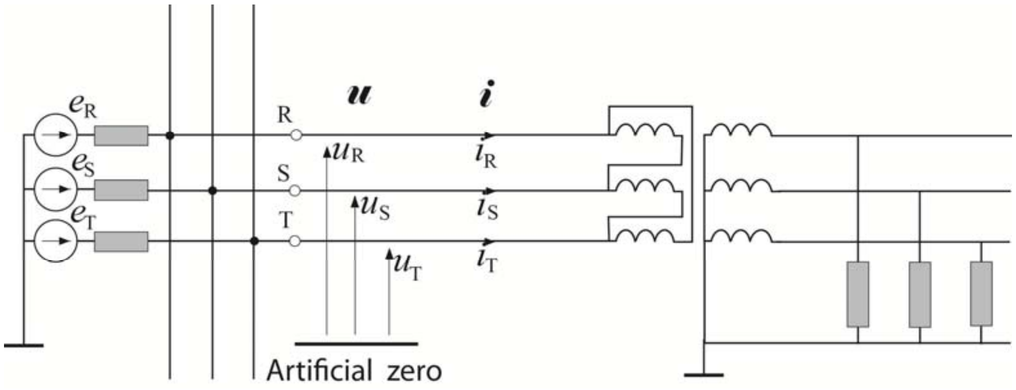
Para wartości chwilowych prądu i napięcia nie dostarcza informacji wystarczającej dla identyfikacji odbiornika

Identyfikacja właściwości energetycznych odbiornika wymaga obserwacji prądu i napięcia w całym okresie zmienności *T*

Moce i Kompensacja w Obwodach z Niesinusoidalnymi Przebiegami Prądu i Napięcia

Cz. 2. Moce w obwodach trójfazowych

Obwód trójfazowy z przebiegami sinusoidalnymi



Moc czynna:

$$P = (\boldsymbol{u}, \boldsymbol{i}) = \sum_{f=\mathrm{R,S,T}} U_f I_f \cos\varphi_f$$

Moc bierna:

$$Q \stackrel{\mathrm{df}}{=} \sum_{f=\mathrm{R,S,T}} U_f I_f \sin\varphi_f$$

Moc pozorna:

$$S_A = U_R I_R + U_S I_S + U_T I_T$$

$$S_G = \sqrt{P^2 + Q^2}$$

$$S_B = \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2}$$

Według której z poniższych definicji należy obliczać moc pozorną w układach trójfazowych?

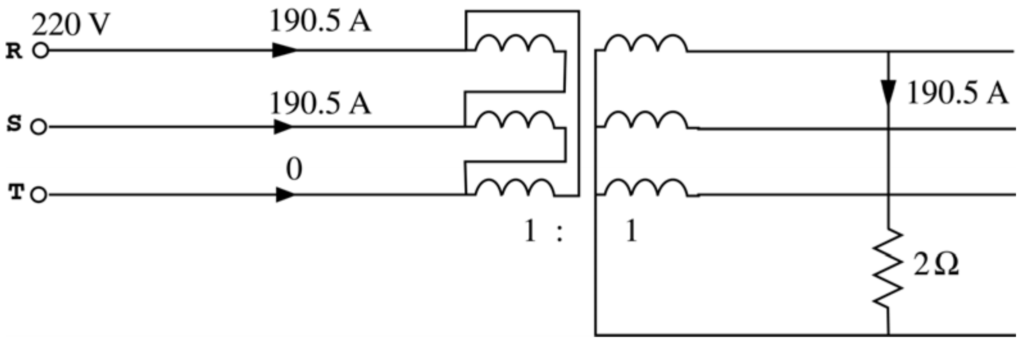
1. $$S_A = U_R I_R + U_S I_S + U_T I_T$$

2. $$S_G = \sqrt{P^2 + Q^2}$$

3. $$S_B = \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2}$$

?

Przykład liczbowy:



P = 72.3 kW

$$S = S_A = U_R I_R + U_S I_S + U_T I_T = 83.8\ \mathrm{kVA}$$

$$S = S_G = \sqrt{P^2 + Q^2} = 72.3\ \mathrm{kVA}$$

$$S = S_B = \sqrt{U_R^2 + U_S^2 + U_T^2}\ \sqrt{I_R^2 + I_S^2 + I_T^2} = 220\sqrt{3} \times 190.2\sqrt{2} = 102.7\ \mathrm{kVA}$$

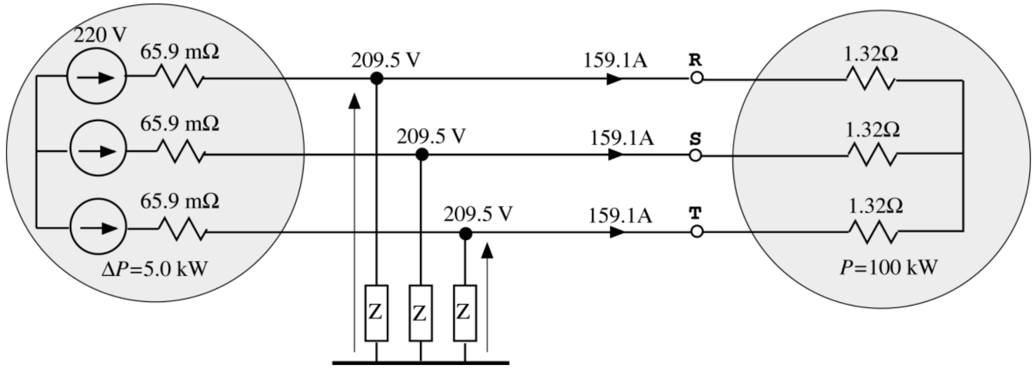
Jaka jest poprawna wartość współczynnika mocy?

$$\lambda = \frac{P}{S}$$

$$\lambda_A = \frac{P}{S_A} = 0.86 \qquad \lambda_G = \frac{P}{S_G} = 1 \qquad \lambda_B = \frac{P}{S_B} = 0.71$$

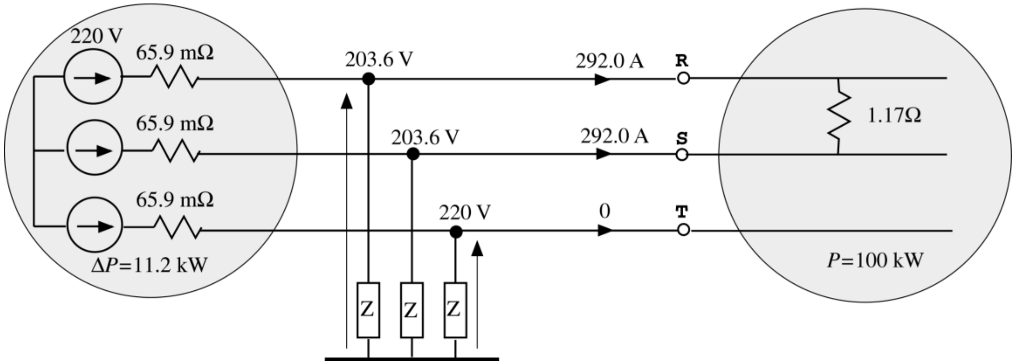
Wybór definicji mocy pozornej:

L.S. Czarnecki: "Energy flow and power phenomena in electrical circuits: illusions and reality," *Archiv fur Elektrotechnik*, (82), No. 4, pp. 10-15, 1999



$S_A = S_G = S_B = 100$ kVA

$\lambda_A = \lambda_G = \lambda_B = 1$

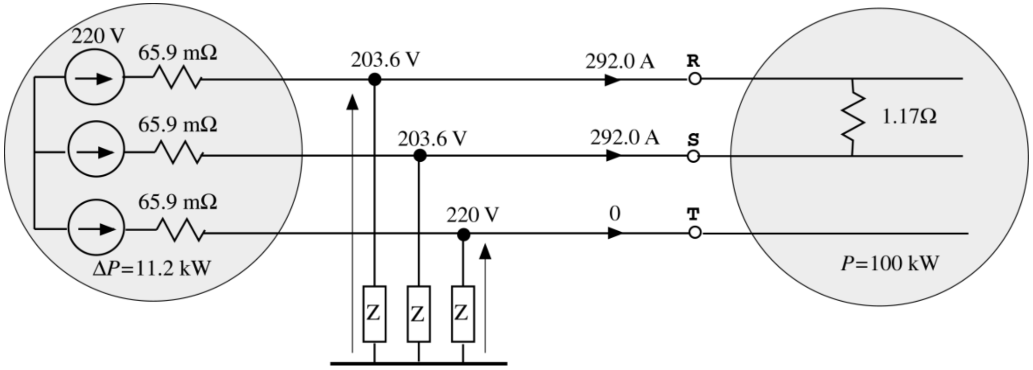


$S_A = 119$ kVA, $\lambda_A = 0.84$

$S_G = 100$ kVA, $\lambda_G = 1$

$S_B = 149$ kVA, $\lambda_B = 0.67$

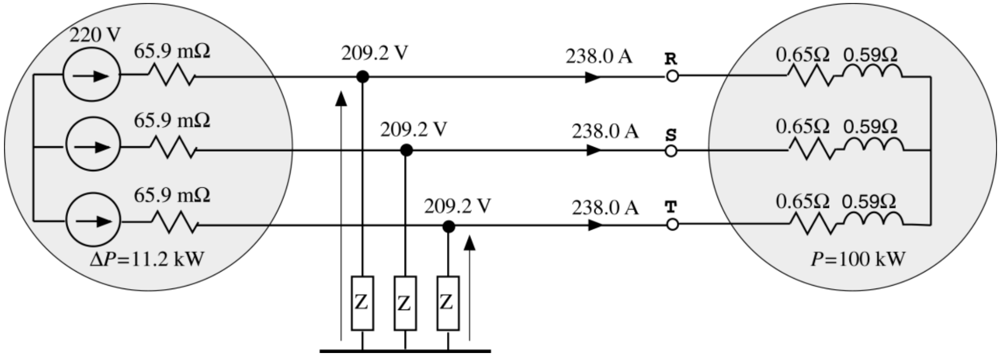
Która z obliczonych wartości mocy pozornej *S* jest poprawna ?



S_A= 119 kVA, λ_A= 0.84

S_G= 100 kVA, λ_G= 1

S_B= 149 kVA, λ_B= 0.67



$S_A = S_G = S_B$=149 kVA

$\lambda_A = \lambda_G = \lambda_B$= 0.67

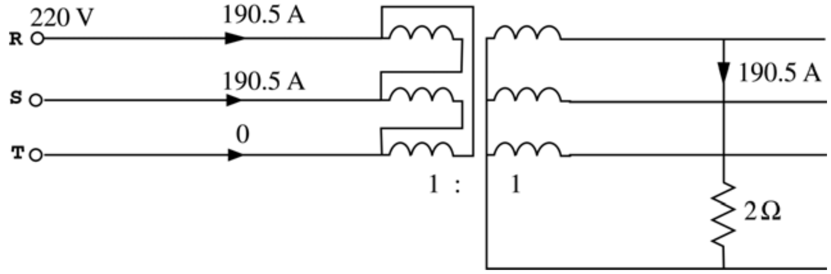
Współczynnik mocy λ jest ze względu na straty mocy w źródle obliczony poprawnie, jeśli

$$S = \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2}$$

Jeśli

$$S = \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2} \qquad S = \sqrt{P^2 + Q^2}$$

Współczynnik mocy λ obliczony jest błędnie



$$P = 72.6\ \text{kW}, \quad Q = 0$$

$$S = S_A \stackrel{\text{df}}{=} U_R I_R + U_S I_S + U_T I_T = 83.8\ \text{kVA}$$

$$S = S_G \stackrel{\text{df}}{=} \sqrt{P^2 + Q^2} = 72.6\ \text{kVA}$$

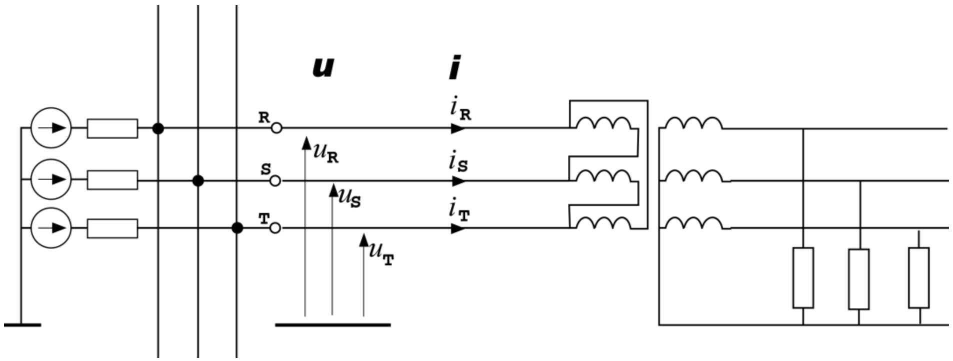
$$S = S_B \stackrel{\text{df}}{=} \sqrt{U_R^2 + U_S^2 + U_T^2}\ \sqrt{I_R^2 + I_S^2 + I_T^2} = 220\sqrt{3} \times 190.2\sqrt{2} = 102.7\ \text{kVA}$$

$$S^2 \neq P^2 + Q^2$$

Prawa strona tej nierówności nie jest obliczona poprawnie

Równanie mocy odbiornika trójfazowego zasilanego trójprzewodowo napięciem sinusoidalnym i symetrycznym

L.S. Czarnecki, Orthogonal decomposition of the current in a three-phase non-linear asymmetrical circuit with nonsinusoidal voltage, *IEEE Trans. Instr. Measur.*, Vol. IM-37, No. 1, **1988**

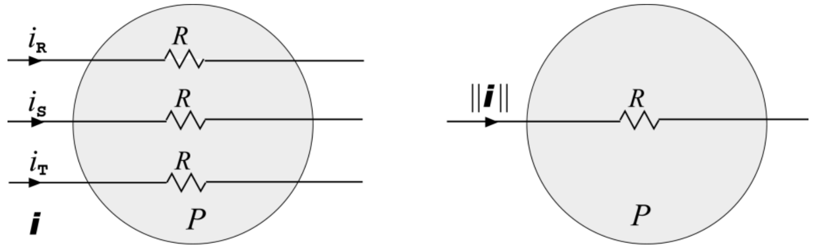


$$\boldsymbol{u}(t) \overset{\text{df}}{=} \begin{bmatrix} u_R(t) \\ u_S(t) \\ u_T(t) \end{bmatrix} \overset{\text{df}}{=} \boldsymbol{u}, \qquad \boldsymbol{i}(t) \overset{\text{df}}{=} \begin{bmatrix} i_R(t) \\ i_S(t) \\ i_T(t) \end{bmatrix} \overset{\text{df}}{=} \boldsymbol{i}$$

$$(\boldsymbol{x}, \boldsymbol{y}) \overset{\text{df}}{=} \frac{1}{T}\int_0^T \boldsymbol{x}^T(t)\, \boldsymbol{y}(t)\, dt$$

$$P = \frac{1}{T}\int_0^T (u_R\, i_R + u_S\, i_S + u_T\, i_T)\, dt = \frac{1}{T}\int_0^T \boldsymbol{u}^T(t)\, \boldsymbol{i}(t)\, dt = (\boldsymbol{u}, \boldsymbol{i})$$

Wartość skuteczna trójfazowego wektora prądów



The active power of a three-phase symmetrical device:

$$P = R\frac{1}{T}\int_0^T (i_R^2 + i_S^2 + i_T^2)\,dt = R\frac{1}{T}\int_0^T \boldsymbol{i}^T(t)\,\boldsymbol{i}(t)\,dt = R(\boldsymbol{i},\boldsymbol{i}) \stackrel{\text{df}}{=} R\,||\boldsymbol{i}||^2$$

$$||\boldsymbol{i}|| \stackrel{\text{df}}{=} \sqrt{(\boldsymbol{i},\boldsymbol{i})}$$

The RMS value of three-phase current ||i|| is a DC current, I, equivalent with respect to active power P to three-phase currents on a symmetrical three-phase device

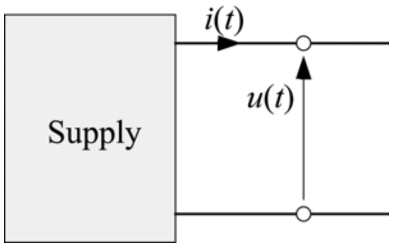
$$||\boldsymbol{i}|| \stackrel{\text{df}}{=} \sqrt{\frac{1}{T}\int_0^T (i_R^2 + i_S^2 + i_T^2)\,dt} = \sqrt{||i_R||^2 + ||i_S||^2 + ||i_T||^2}$$

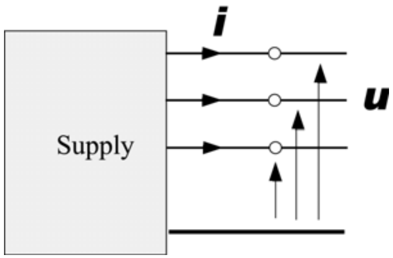
is the RMS value of the three-phase current

$$||\boldsymbol{u}|| \stackrel{\text{df}}{=} \sqrt{\frac{1}{T}\int_0^T (u_R^2 + u_S^2 + u_T^2)\,dt} = \sqrt{||u_R||^2 + ||u_S||^2 + ||u_T||^2}$$

is the RMS value of the three-phase voltage

Definicja mocy pozornej *S*





Moc pozorna jest wielością umowną:

$$S \stackrel{\text{df}}{=} ||u||\,||i||$$

$$S \stackrel{\text{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}||$$

Moc pozorna *S* jest iloczynem wartości skutecznych prądu i napięcia potrzebnych do zasilania odbiornika

Pod tym względem nie ma różnicy między odbiorniem jednofazowym i trójfazowym

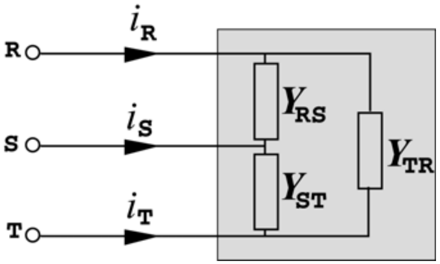
Przy sinusoidalnych przebiegach prądu i napięcia, prowadzi to do definicji Buchholza:

$$S \stackrel{\text{df}}{=} \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2}$$

Rozkład sinusoidalnego prądu trójfazowego na składowe fizyczne

L.S. Czarnecki: “Equivalent circuits of unbalanced loads supplied with symmetrical and asymmetrical voltage and their identification”, *Archiv fur Elektrotechnik,* 78 pp. 165-168, 1995

$$\boldsymbol{u} \overset{\text{df}}{=} \begin{bmatrix} u_{\rm R} \\ u_{\rm S} \\ u_{\rm T} \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{U}_{\rm R} \\ \boldsymbol{U}_{\rm S} \\ \boldsymbol{U}_{\rm T} \end{bmatrix} e^{j\omega_1 t} \overset{\text{df}}{=} \sqrt{2}\,\mathrm{Re}\, \boldsymbol{U}\, e^{j\omega_1 t}$$



$$\boldsymbol{i} \overset{\text{df}}{=} \begin{bmatrix} i_{\rm R} \\ i_{\rm S} \\ i_{\rm T} \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{I}_{\rm R} \\ \boldsymbol{I}_{\rm S} \\ \boldsymbol{I}_{\rm T} \end{bmatrix} e^{j\omega_1 t} \overset{\text{df}}{=} \sqrt{2}\,\mathrm{Re}\, \boldsymbol{I}\, e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\{(\boldsymbol{Y}_{\rm e} \boldsymbol{U} + \boldsymbol{A}\, \boldsymbol{U}^{\#})\, e^{j\omega_1 t}\}$$

$$\begin{bmatrix} \boldsymbol{U}_{\rm R} \\ \boldsymbol{U}_{\rm T} \\ \boldsymbol{U}_{\rm S} \end{bmatrix} \overset{\text{df}}{=} \boldsymbol{U}^{\#}$$

$$\boldsymbol{Y}_{\rm RS} + \boldsymbol{Y}_{\rm ST} + \boldsymbol{Y}_{\rm TR} \overset{\text{df}}{=} \boldsymbol{Y}_{\rm e},$$ Admitancja równoważna

$$-(\boldsymbol{Y}_{\rm ST} + \alpha \boldsymbol{Y}_{\rm TR} + \alpha^{*} \boldsymbol{Y}_{\rm RS}) \overset{\text{df}}{=} \boldsymbol{A},$$ Admitancja niezrównoważenia

$$\boldsymbol{i} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_{\mathrm{e}}\,\boldsymbol{U} + \boldsymbol{A}\,\boldsymbol{U}^{\#}\}\,e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\} + \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\} + \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\,\boldsymbol{U}^{\#}e^{j\omega_1 t}\}$$

$$\boldsymbol{i}_{\mathrm{a}} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\}$$ Prąd czynny

$$\boldsymbol{i}_{\mathrm{r}} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\}$$ Prąd bierny

$$\boldsymbol{i}_{\mathrm{u}} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\,\boldsymbol{U}^{\#}e^{j\omega_1 t}\}$$ Prąd niezrównoważenia

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}$$

Są to Składowe Fizyczne Prądu (Currents Physical Components-CPC)
liniowego odbiornika trójfazowego
zasilanego napięciem sinusoidalnym i symetrycznym

<u>Iloczyn skalarny wektorów trójfazowych</u>

$$(\boldsymbol{x}, \boldsymbol{y}) \overset{\text{df}}{=} \frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}\, dt = \frac{1}{T}\int_0^T (x_{\mathrm{R}} y_{\mathrm{R}} + x_{\mathrm{S}} y_{\mathrm{S}} + x_{\mathrm{T}} y_{\mathrm{T}})\, dt = (x_{\mathrm{R}}, y_{\mathrm{R}}) + (x_{\mathrm{S}}, y_{\mathrm{S}}) + (x_{\mathrm{T}}, y_{\mathrm{T}}) =$$

$$= \mathrm{Re}\{\boldsymbol{X}_{\mathrm{R}}\boldsymbol{Y}_{\mathrm{R}}^{*}\} + \mathrm{Re}\{\boldsymbol{X}_{\mathrm{S}}\boldsymbol{Y}_{\mathrm{S}}^{*}\} + \mathrm{Re}\{\boldsymbol{X}_{\mathrm{T}}\boldsymbol{Y}_{\mathrm{T}}^{*}\} = \mathrm{Re}\{X^{\mathrm{T}} Y^{*}\}$$

Wektory $\boldsymbol{x}(t)$ i $\boldsymbol{y}(t)$ są wzajemnie ortogonalne wtedy,

gdy ich iloczyn skalarny $(\boldsymbol{x}, \boldsymbol{y}) = 0$

Wówczas:

$$\|\boldsymbol{x} + \boldsymbol{y}\|^2 = \|\boldsymbol{x}\|^2 + \|\boldsymbol{y}\|^2$$

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}$$

Prady czynny, bierny i niezrównoważenia są wzajemnie ortogonalne

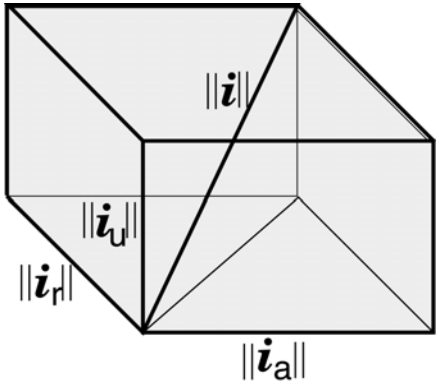
$$(\boldsymbol{i}_{\mathrm{a}}, \boldsymbol{i}_{\mathrm{r}}) = 0, \quad (\boldsymbol{i}_{\mathrm{a}}, \boldsymbol{i}_{\mathrm{u}}) = 0, \quad (\boldsymbol{i}_{\mathrm{r}}, \boldsymbol{i}_{\mathrm{u}}) = 0,$$

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2$$

$$||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{e}} \, ||\boldsymbol{u}||$$

$$||\boldsymbol{i}_{\mathrm{u}}|| = A \, ||\boldsymbol{u}||$$

$$||\boldsymbol{i}_{\mathrm{r}}|| = |B_{\mathrm{e}}| \, ||\boldsymbol{u}||$$



Równanie mocy liniowego odbiornika trójfazowego zasilanego trójprzewodowo symetrycznym i sinusoidalnym napięciem

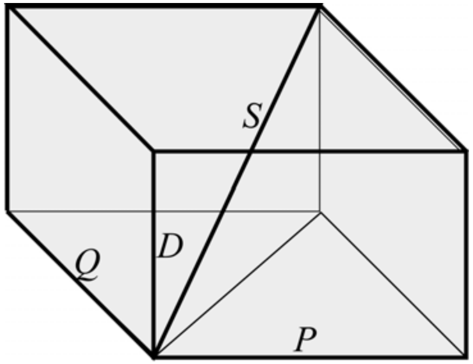
$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2 \qquad | \times ||\boldsymbol{u}||^2$$

$$S^2 = P^2 + Q^2 + D_u^2$$

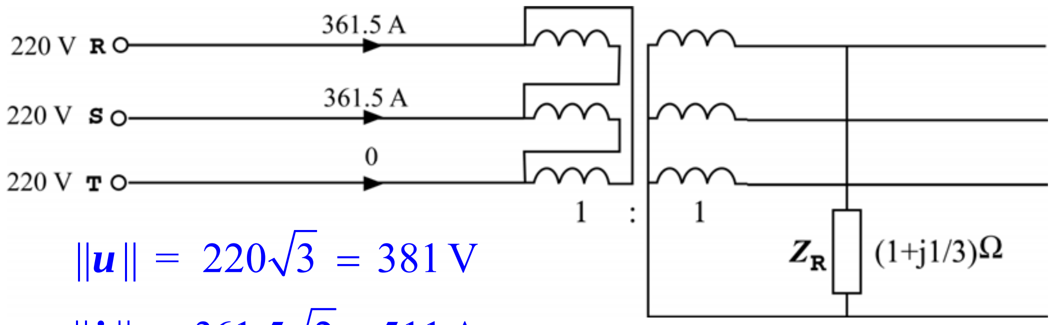
$$P \overset{\mathrm{df}}{=} ||\boldsymbol{i}_a||\, ||\boldsymbol{u}|| = G_e\, ||\boldsymbol{u}||^2$$ Moc czynna

$$Q \overset{\mathrm{df}}{=} \pm\, ||\boldsymbol{i}_r||\, ||\boldsymbol{u}|| = -|B_e|\, ||\boldsymbol{u}||^2$$ Moc bierna

$$D_u \overset{\mathrm{df}}{=} ||\boldsymbol{i}_u||\, ||\boldsymbol{u}|| = A\, ||\boldsymbol{u}||^2$$ Moc niezrównoważenia



Numerical illustration



$$\|\boldsymbol{u}\| = 220\sqrt{3} = 381\ \text{V}$$

$$\|\boldsymbol{i}\| = 361.5\sqrt{2} = 511\ \text{A}$$

$$\boldsymbol{Y}_{RS} = \frac{1}{\boldsymbol{Z}_R} = 0.90 - j\,0.30 = 0.95\,e^{-j18^0}\ \text{S}$$

$$\boldsymbol{Y}_e = G_e + jB_e = \boldsymbol{Y}_{RS} = 0.90 - j\,0.30\ \text{S}$$

$$\boldsymbol{A} = -\alpha^{*}\boldsymbol{Y}_{RS} = 0.95\,e^{j42^0}\ \text{S}$$

$$\|\boldsymbol{i}_a\| = G_e\|\boldsymbol{u}\| = 0.90 \times 381 = 343\ \text{A}$$

$$\|\boldsymbol{i}_r\| = |B_e|\|\boldsymbol{u}\| = 0.30 \times 381 = 114\ \text{A}$$

$$\|\boldsymbol{i}_u\| = A\|\boldsymbol{u}\| = 0.95 \times 381 = 361\ \text{A}$$

$$\|\boldsymbol{i}\| = \sqrt{\|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_r\|^2 + \|\boldsymbol{i}_u\|^2} = \sqrt{343^2 + 114^2 + 361^2} = 511\ \text{A}$$

$S = 195$ kVA, $P = 131$ kW, $Q = 43$ kVAr, $D_u = 138$ kVA

Moce chwilowe liniowego odbiornika trójfazowego:

$$\frac{dW(t)}{dt} = p(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}) \overset{\mathrm{df}}{=} p_{\mathrm{a}}(t) + p_{\mathrm{r}}(t) + p_{\mathrm{u}}(t)$$

$$p_{\mathrm{a}}(t) \overset{\mathrm{df}}{=} \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{a}} = \boldsymbol{u}^{\mathrm{T}} G_{\mathrm{e}} \boldsymbol{u} = 3 G_{\mathrm{e}} U^2 = P = \mathrm{const.}$$

$$p_{\mathrm{r}}(t) \overset{\mathrm{df}}{=} \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{r}} \equiv 0.$$

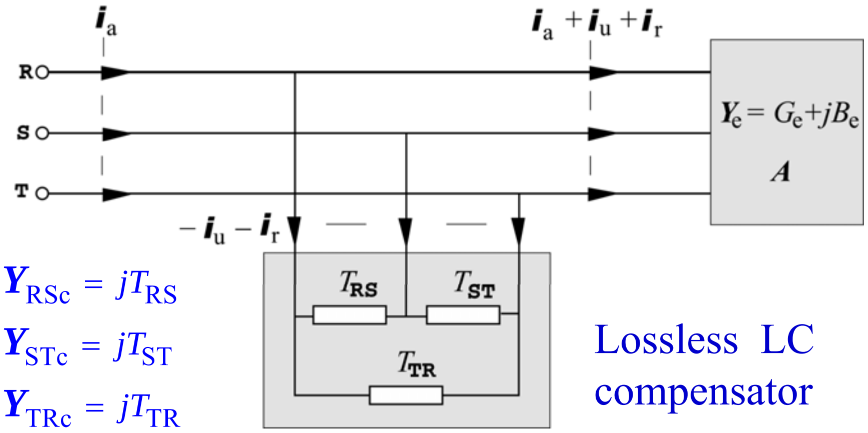
$$p_{\mathrm{u}}(t) \overset{\mathrm{df}}{=} \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{u}} = -3AU^2\cos(2\omega_1 t + \psi) = -D\cos(2\omega_1 t + \psi)$$

$$\frac{dW(t)}{dt} = p(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = P - D\cos(2\omega_1 t + \psi)$$

Energy oscillation in three-phase systems
with sinusoidal voltages and currents
can occur only because of the load current asymmetry

Compensation of the reactive and unbalanced currents

$$\lambda \stackrel{\text{df}}{=} \frac{P}{S} = \frac{||\boldsymbol{i}_a||}{\sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_u||^2 + ||\boldsymbol{i}_r||^2}}$$



$\boldsymbol{Y}_{RSc} = jT_{RS}$

$\boldsymbol{Y}_{STc} = jT_{ST}$

$\boldsymbol{Y}_{TRc} = jT_{TR}$

Lossless LC compensator

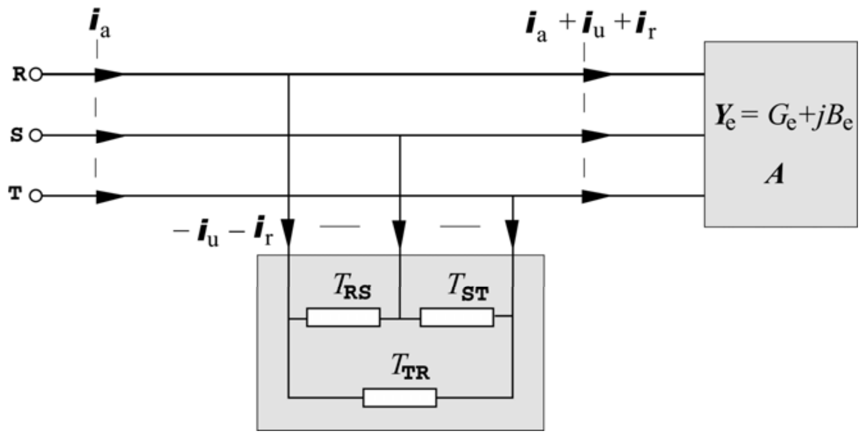
$$\boldsymbol{i}_r' = \sqrt{2}\,\mathrm{Re}\{\, j[B_e + (T_{ST} + T_{TR} + T_{RS})]\, \boldsymbol{U}\,\} e^{j\omega_1 t}$$

$$\boldsymbol{i}_u' = \sqrt{2}\,\mathrm{Re}\{[\boldsymbol{A} - j(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS})]\, \boldsymbol{U}^{\#}\} e^{j\omega_1 t}$$

The reactive & unbalanced currents are compensated totally, if

$$B_e + (T_{ST} + T_{TR} + T_{RS}) = 0 \qquad (1)$$

$$\boldsymbol{A} - j(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS}) = 0 \qquad (2)\ \&\ (3)$$



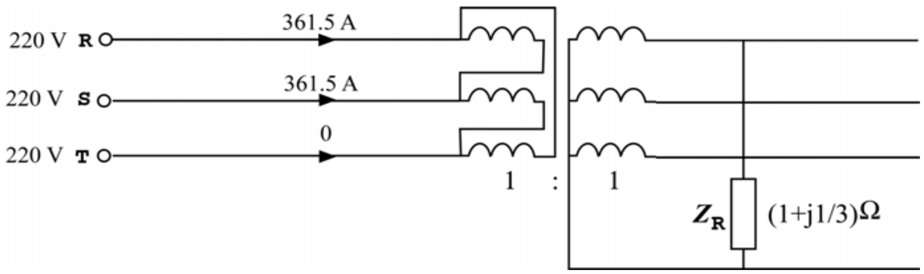
The reactive & unbalanced currents are compensated totally, if

$$T_{RS} = (\sqrt{3}\,\mathrm{Re}\{\boldsymbol{A}\} - \mathrm{Im}\{\boldsymbol{A}\} - B_e)/3$$

$$T_{ST} = (2\,\mathrm{Im}\,\{\boldsymbol{A}\} - B_e)/3$$

$$T_{TR} = (-\sqrt{3}\,\mathrm{Re}\{\boldsymbol{A}\} - \mathrm{Im}\{\boldsymbol{A}\} - B_e)/3$$

Numerical illustration



Load parameters:

$$\boldsymbol{Y}_e = G_e + jB_e = \boldsymbol{Y}_{RS} = 0.90 - j\,0.30\ \mathrm{S}$$

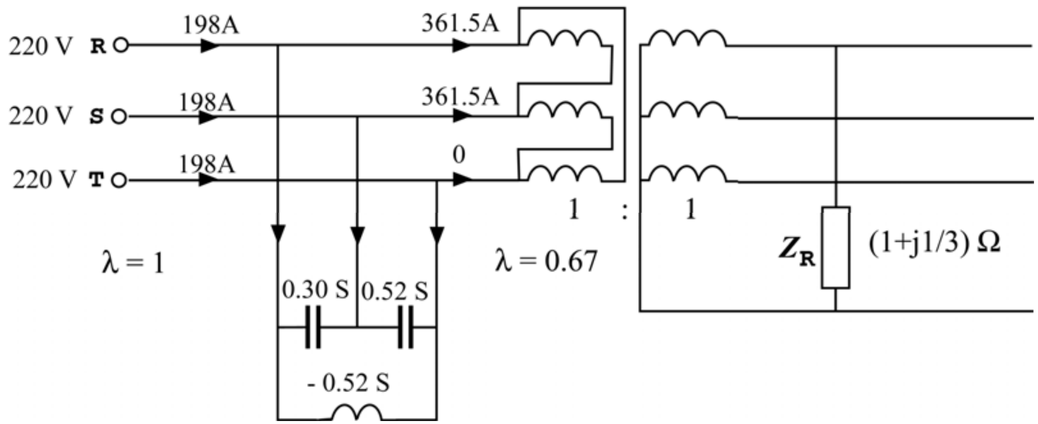
$$\boldsymbol{A} = -\alpha^* \boldsymbol{Y}_{RS} = 0.95\, e^{j\,42^0} = 0.71 + j\,0.64\ \mathrm{S}$$

$$||\boldsymbol{i}_a|| = 343\ \mathrm{A},\quad ||\boldsymbol{i}_u|| = 361\ \mathrm{A},\quad ||\boldsymbol{i}_r|| = 114\ \mathrm{A},\quad ||\boldsymbol{i}|| = 511\ \mathrm{A},\quad S = 195\ \mathrm{kVA},\quad \lambda = 0.67$$

$$T_{RS} = (\sqrt{3}\,\mathrm{Re}\{\boldsymbol{A}\} - \mathrm{Im}\{\boldsymbol{A}\} - B_e)/3 = 0.30\ \mathrm{S}$$

$$T_{ST} = (2\,\mathrm{Im}\{\boldsymbol{A}\} - B_e)/3 = 0.52\ \mathrm{S}$$

$$T_{TR} = (-\sqrt{3}\,\mathrm{Re}\{\boldsymbol{A}\} - \mathrm{Im}\{\boldsymbol{A}\} - B_e)/3 = -0.52\ \mathrm{S}$$



$$||\boldsymbol{i}_a|| = 343\ \mathrm{A},\quad ||\boldsymbol{i}_u|| = 0,\quad ||\boldsymbol{i}_r|| = 0,\quad ||\boldsymbol{i}|| = 343\ \mathrm{A},\quad S = 131\ \mathrm{kVA},\quad \lambda = 1$$

CPC – based power theory of three-phase systems with LTI loads with symmetrical nonsinusoidal voltages

Condition: $n \neq 3k$

$$\boldsymbol{u} = \sum_{n\in N} \boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_n\, e^{jn\omega_1 t}$$

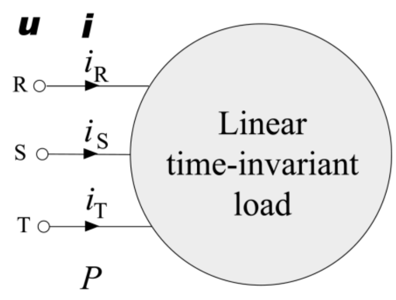
$$\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} \overset{\mathrm{df}}{=} \boldsymbol{U}_n \qquad \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{T}n} \\ \boldsymbol{U}_{\mathrm{S}n} \end{bmatrix} \overset{\mathrm{df}}{=} \boldsymbol{U}_n^{\#}$$

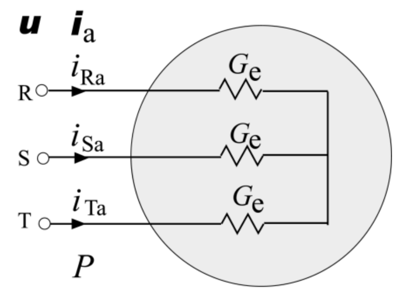
$$\boldsymbol{i}_n = \sqrt{2}\,\mathrm{Re}\,\boldsymbol{I}_n\, e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\{(G_{\mathrm{e}n}\boldsymbol{U}_n + jB_{\mathrm{e}n}\boldsymbol{U}_n + \boldsymbol{A}_n \boldsymbol{U}_n^{\#})\, e^{jn\omega_1 t}\} = \boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}$$

$$\boldsymbol{i}_{\mathrm{a}n} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}n}\boldsymbol{U}_{\boldsymbol{n}}\, e^{jn\omega_1 t}\}$$

$$\boldsymbol{i}_{\mathrm{r}n} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{e}n}\boldsymbol{U}_{\boldsymbol{n}}\, e^{jn\omega_1 t}\}$$

$$\boldsymbol{i}_{\mathrm{u}n} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}_n \boldsymbol{U}_n^{\#}\, e^{jn\omega_1 t}\}$$





Active current: $\boldsymbol{i}_a \overset{df}{=} G_e \boldsymbol{u}, \qquad G_e \overset{df}{=} \frac{P}{\|\boldsymbol{u}\|^2}$

$$\boldsymbol{i} - \boldsymbol{i}_a = \sum_{n\in N} \boldsymbol{i}_n - \boldsymbol{i}_a = \sum_{n\in N} (\boldsymbol{i}_{an} + \boldsymbol{i}_{rn} + \boldsymbol{i}_{un}) - \boldsymbol{i}_a = (\sum_{n\in N} \boldsymbol{i}_{an} - \boldsymbol{i}_a) + \sum_{n\in N} \boldsymbol{i}_{rn} + \sum_{n\in N} \boldsymbol{i}_{un}$$

$$\boldsymbol{i}_s \overset{df}{=} \sum_{n\in N} \boldsymbol{i}_{an} - \boldsymbol{i}_a = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} (G_{en} - G_e)\, \boldsymbol{U}_n\, e^{jn\omega_1 t}$$ Scattered current

$$\boldsymbol{i}_r \overset{df}{=} \sum_{n\in N} \boldsymbol{i}_{rn} = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} jB_{en}\, \boldsymbol{U}_n\, e^{jn\omega_1 t}$$ Reactive current

$$\boldsymbol{i}_u \overset{df}{=} \sum_{n\in N} \boldsymbol{i}_{un} = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} A_n\, \boldsymbol{U}_n^{\#}\, e^{jn\omega_1 t}$$ Unbalanced current

Decomposition three-phase current into CPC:

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_s + \boldsymbol{i}_r + \boldsymbol{i}_u$$

The active, scattered, reactive and unbalanced currents are mutually orthogonal

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{s}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2$$

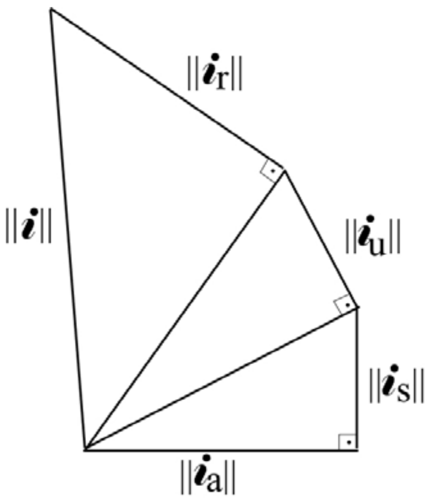
The RMS values of the current components:

$$||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{e}}\, ||\boldsymbol{u}||$$

$$||\boldsymbol{i}_{\mathrm{s}}|| = \sqrt{\sum_{n \in N} (G_{\mathrm{e}n} - G_{\mathrm{e}})^2\, ||\boldsymbol{u}_n||^2}$$

$$||\boldsymbol{i}_{\mathrm{u}}|| = \sqrt{\sum_{n \in N} A_n\, ||\boldsymbol{u}_n||^2}$$

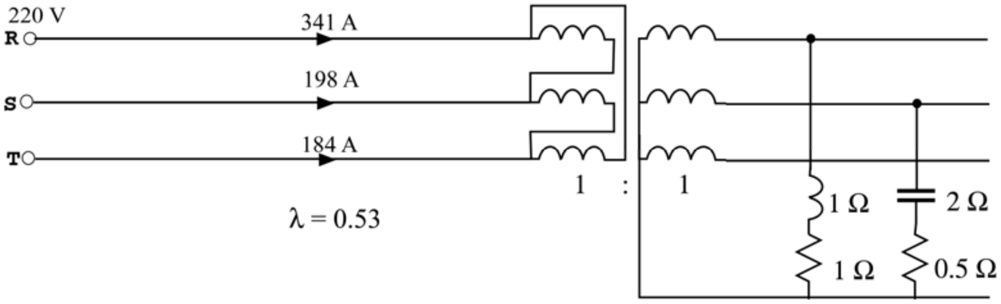
$$||\boldsymbol{i}_{\mathrm{r}}|| = \sqrt{\sum_{n \in N} |\boldsymbol{B}_n|^2\, ||\boldsymbol{u}_n||^2}$$



Observe the difference:

$$G_{\mathrm{e}} \overset{\mathrm{df}}{=} \frac{P}{||\boldsymbol{u}||^2}, \qquad G_{\mathrm{e}n} \overset{\mathrm{df}}{=} \frac{P_n}{||\boldsymbol{u}_n||^2}$$

$$u_R(t) = \sqrt{2}\,\mathrm{Re}\{220\,e^{j\omega_1 t} + 44\,e^{j5\omega_1 t}\}\ \mathrm{V}$$



$$G_e = 0.6018\ \mathrm{S}$$
$$\boldsymbol{Y}_{e1} = 0.60 - j0.40\ \mathrm{S}$$
$$\boldsymbol{Y}_{e5} = 0.88 + j0.15\ \mathrm{S}$$
$$\boldsymbol{A}_1 = 0.83\ e^{-j0.18\pi}\ \mathrm{S}$$
$$\boldsymbol{A}_5 = 1.12\ e^{-j0.86\pi}\ \mathrm{S}$$

$$||\boldsymbol{i}\,|| = \sqrt{||i_R||^2 + ||i_S||^2 + ||i_T||^2} = \sqrt{341^2 + 198^2 + 184^2} = 433\ \mathrm{A}$$

$$||\boldsymbol{i}_a|| = 237\ \mathrm{A}$$
$$||\boldsymbol{i}_s|| = 21\ \mathrm{A}$$
$$||\boldsymbol{i}_r|| = 153\ \mathrm{A}$$
$$||\boldsymbol{i}_u|| = 327\ \mathrm{A}$$

Verification:

$$||\boldsymbol{i}\,|| = \sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2} = \sqrt{237^2 + 21^2 + 153^2 + 327^2} = 433\ \mathrm{A}$$

Powers of three-phase LTI loads supplied with nonsinusoidal voltage

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{s}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 \qquad | \times ||\boldsymbol{u}||^2$$

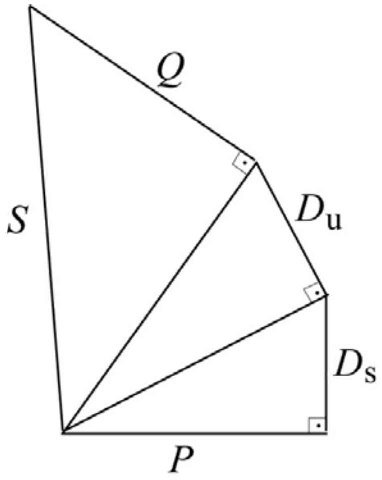
$$S^2 = P^2 + D_{\mathrm{s}}^2 + D_{\mathrm{u}}^2 + Q^2$$

$$P \overset{\mathrm{df}}{=} ||\boldsymbol{i}_{\mathrm{a}}||\,||\boldsymbol{u}|| = G_{\mathrm{e}}||\boldsymbol{u}||^2$$

$$D_{\mathrm{s}} \overset{\mathrm{df}}{=} ||\boldsymbol{i}_{\mathrm{s}}||\,||\boldsymbol{u}||$$

$$D_{\mathrm{u}} \overset{\mathrm{df}}{=} ||\boldsymbol{i}_{\mathrm{u}}||\,||\boldsymbol{u}||$$

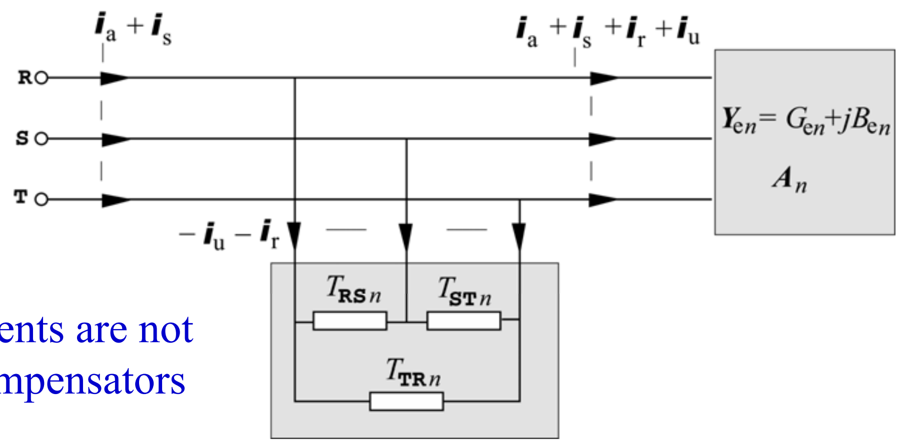
$$Q \overset{\mathrm{df}}{=} ||\boldsymbol{i}_{\mathrm{r}}||\,||\boldsymbol{u}||$$



$$\lambda \overset{\mathrm{df}}{=} \frac{P}{S} = \frac{||\boldsymbol{i}_{\mathrm{a}}||}{\sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{s}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2}}$$

Compensation of the reactive and unbalanced currents

L.S. Czarnecki: *Reactive and unbalanced currents compensation in three-phase circuits under nonsinusoidal conditions,* IEEE Trans. Instr. Measur., Vol. IM-38, No. 3, June 1989.



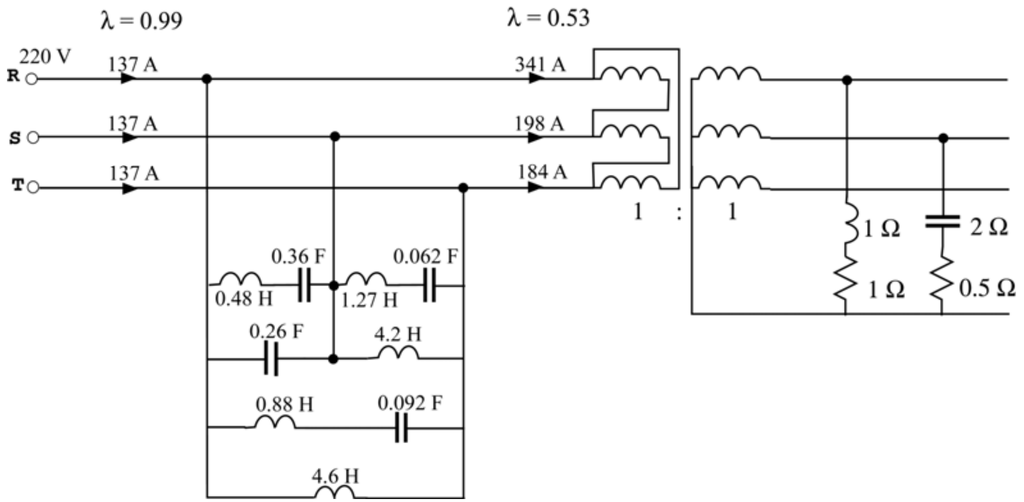
Active & scattered currents are not affected by reactive compensators

The reactive & unbalanced currents are compensated for each harmonic, if

$$B_{en} + (T_{STn} + T_{TRn} + T_{RSn}) = 0$$

$$\boldsymbol{A}_n - j\,(T_{STn} + \beta T_{TRn} + \beta^* T_{RSn}) = 0, \qquad \beta \overset{\text{df}}{=} \begin{cases} \alpha & -\text{pos. sequence} \\ \alpha^* & -\text{neg. sequence} \end{cases}$$

From these equations the susceptances T_{RSn}, T_{STn}, and T_{TRn}, can be calculated



$G_e = 0.6018$ S

$\boldsymbol{Y}_{e1} = 0.60 - j0.40$ S

$\boldsymbol{Y}_{e5} = 0.88 + j0.15$ S

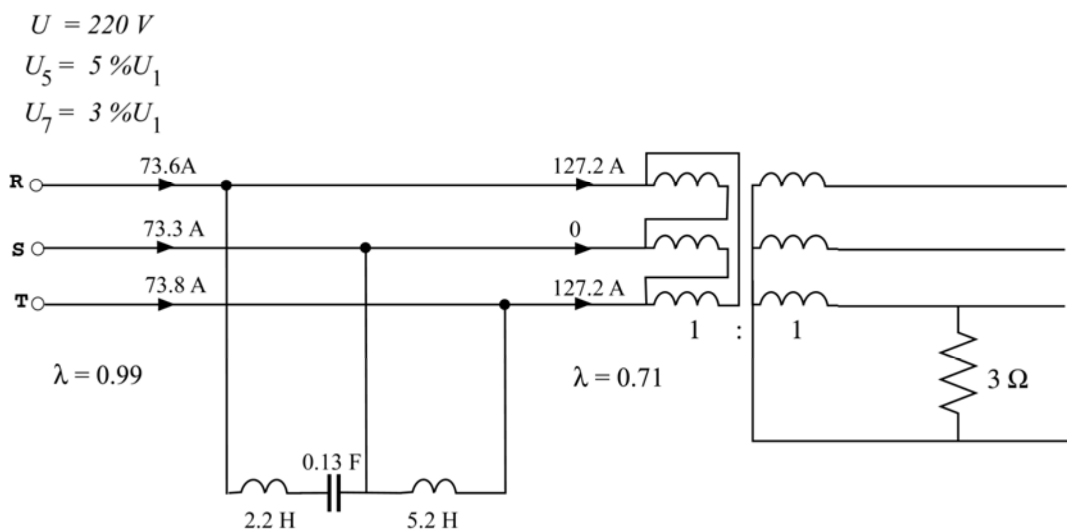
$\boldsymbol{A}_1 = 0.83\, e^{-j0.18\pi}$ S

$\boldsymbol{A}_5 = 1.12\, e^{-j0.86\pi}$ S

$\|\|\boldsymbol{i}_a\|\| = 237$ A	$\|\|\boldsymbol{i}_a\|\| = 237$ A
$\|\|\boldsymbol{i}_s\|\| = 21$ A	$\|\|\boldsymbol{i}_s\|\| = 21$ A
$\|\|\boldsymbol{i}_r\|\| = 0$	$\|\|\boldsymbol{i}_r\|\| = 153$ A
$\|\|\boldsymbol{i}_u\|\| = 0$	$\|\|\boldsymbol{i}_u\|\| = 327$ A
$\|\|\boldsymbol{i}\|\| = 238$ A	$\|\|\boldsymbol{i}\|\| = 433$ A

This is example **of total compensation** of the reactive and unbalanced currents

L.S. Czarnecki, *Minimization of unbalanced and reactive currents in three-phase asymmetrical circuits with nonsinusoidal voltage,* *Proc. IEE,* Vol. 139, Pt. B, 1992



$G_e = 0.33$ S

$\boldsymbol{Y}_{e1} = \boldsymbol{Y}_{e5} = \boldsymbol{Y}_{e7} = 0.33$ S

$\boldsymbol{A}_1 = \boldsymbol{A}_7 = 0.33\, e^{j60}$ S

$\boldsymbol{A}_5 = 0.33\, e^{-j60}$ S

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_r + \boldsymbol{i}_u$$

$||\boldsymbol{i}_a|| = 127.2$ A

$||\boldsymbol{i}_s|| = 0$

$||\boldsymbol{i}_r|| = 2.9$ A

$||\boldsymbol{i}_u|| = 6.9$ A

$||\boldsymbol{i}|| = 127.4$ A

$||\boldsymbol{i}_a|| = 127.2$ A

$||\boldsymbol{i}_s|| = 0$

$||\boldsymbol{i}_r|| = 0$

$||\boldsymbol{i}_u|| = 127.2$ A

$||\boldsymbol{i}|| = 179.9$ A

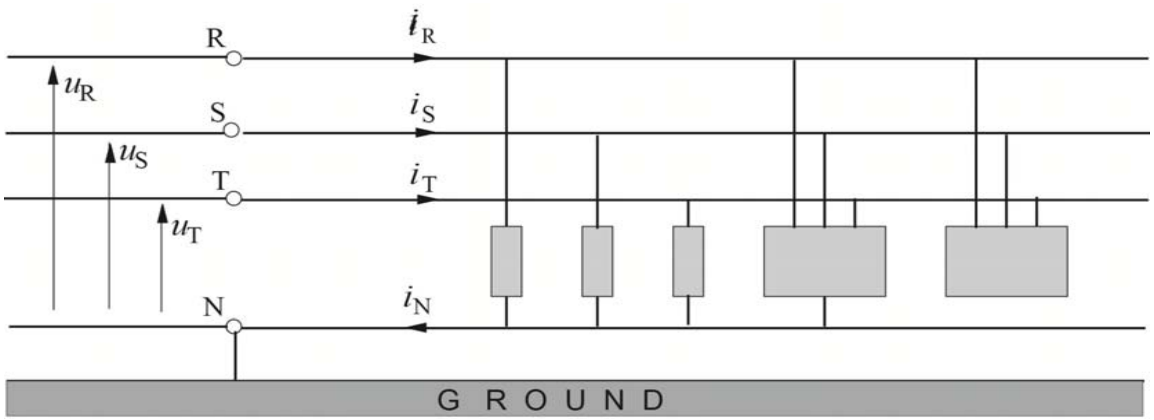
$T_{RS1} = -0.19$ S, $T_{RS5} = 0.19$ S, $T_{RS7} = -0.19$ S

$T_{ST1} = 0.19$ S, $T_{ST5} = -0.19$ S, $T_{ST7} = 0.19$ S

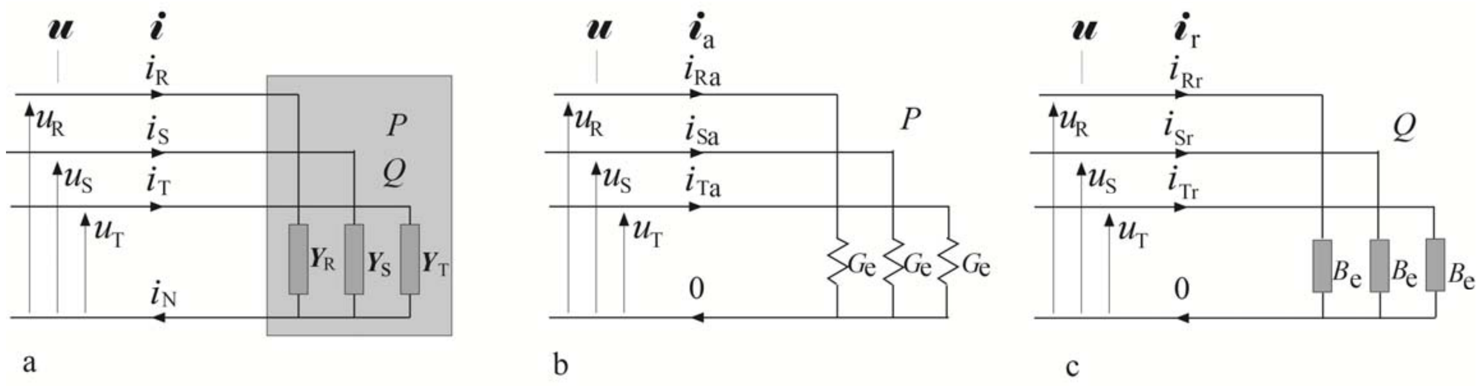
$T_{TR} = 0$

Powyższe wyniki dotyczą jednak jedynie odbiorników trójfazowych zasilanych trójprzewodowo

Nie są one prawdziwe w przypadków odbiorników trójfazowych z przewodem zerowym



Odbiorniki równoważne ze względu na moc czynną i moc bierną



$$\boldsymbol{i}_{\mathrm{a}}(t) = G_{\mathrm{e}}\, \boldsymbol{u}(t) \qquad G_{\mathrm{e}} = \frac{P}{\|\boldsymbol{u}\|^2} = \frac{P}{3U_{\mathrm{R}}^2} = \frac{1}{3}(G_{\mathrm{R}} + G_{\mathrm{S}} + G_{\mathrm{T}})$$

$$\boldsymbol{i}_{\mathrm{r}}(t) = B_{\mathrm{e}} \frac{d}{d(\omega t)} \boldsymbol{u}(t)\,. \qquad B_{\mathrm{e}} = -\frac{Q}{\|\boldsymbol{u}\|^2} = -\frac{1}{3}\frac{Q}{U_{\mathrm{R}}^2} = \frac{1}{3}(B_{\mathrm{R}} + B_{\mathrm{S}} + B_{\mathrm{T}})$$

$$\boldsymbol{i} - \boldsymbol{i}_{\mathrm{a}} - \boldsymbol{i}_{\mathrm{r}} = \boldsymbol{i}_{\mathrm{u}} = \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$$

,
,
,

Składowe Fizyczne Prądu zasilania

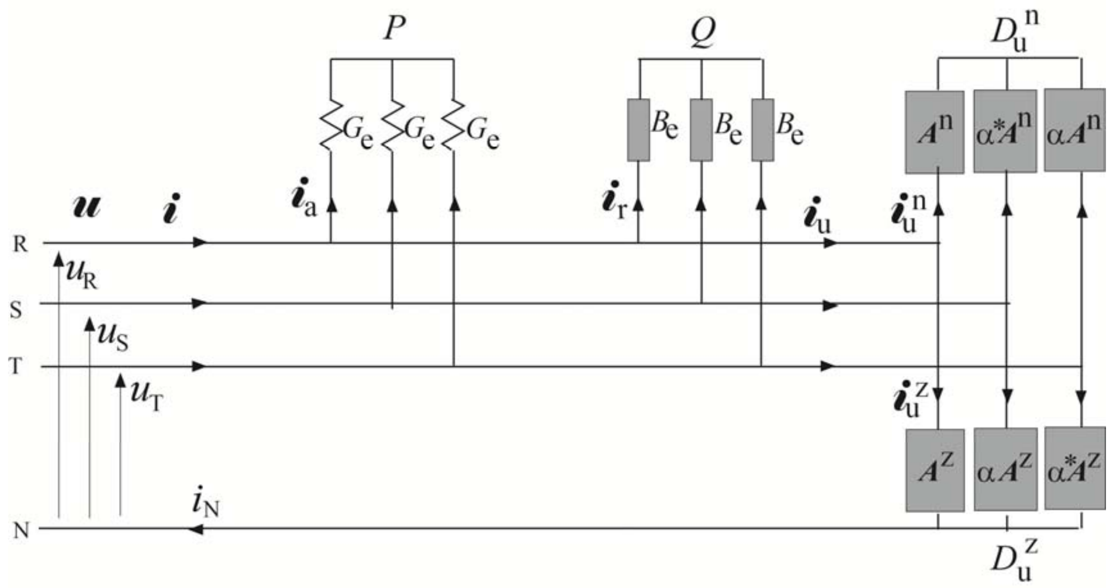
$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$$

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} \stackrel{\mathrm{df}}{=} \sqrt{2}\mathrm{Re}\left\{\begin{bmatrix} \boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{T}} \\ \boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{S}} \end{bmatrix} e^{j\omega t}\right\} = \sqrt{2}\mathrm{Re}\{\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\#}e^{j\omega t}\}$$

$$\boldsymbol{A}^{\mathrm{n}} \stackrel{\mathrm{df}}{=} \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}} + \alpha \boldsymbol{Y}_{\mathrm{S}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{T}})$$

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}} \stackrel{\mathrm{df}}{=} \sqrt{2}\mathrm{Re}\left\{\begin{bmatrix} \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}_{\mathrm{R}} \end{bmatrix} e^{j\omega t}\right\} = \sqrt{2}\mathrm{Re}\{\boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}_{\mathrm{R}}e^{j\omega t}\}.$$

$$\boldsymbol{A}^{\mathrm{z}} \stackrel{\mathrm{df}}{=} \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{S}} + \alpha \boldsymbol{Y}_{\mathrm{T}}).$$



$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$$

Składowe Fizyczne Prądu są wzajemnie ortogonalne, zatem

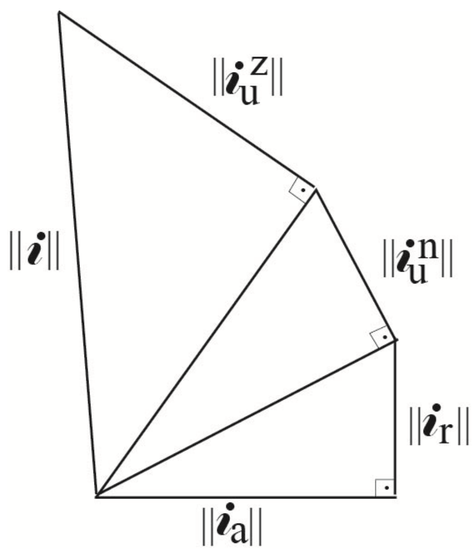
$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}||^2.$$

$$||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{e}}\,||\boldsymbol{u}||$$

$$||\boldsymbol{i}_{\mathrm{r}}|| = |B_{\mathrm{e}}|\,||\boldsymbol{u}||$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}|| = A^{\mathrm{n}}\,||\boldsymbol{u}||$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}|| = A^{\mathrm{z}}\,||\boldsymbol{u}||.$$



$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{n}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{z}\|^2 \qquad | \times \|\boldsymbol{u}\|^2$$

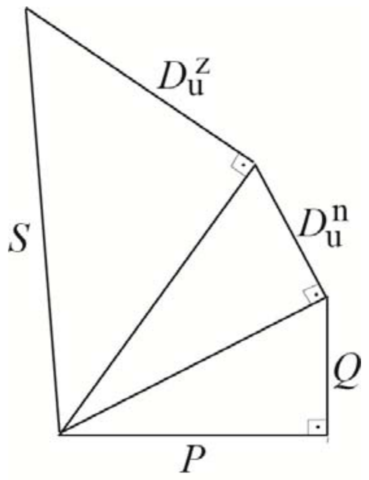
$$S^2 = P^2 + Q^2 + D_\mathrm{u}^{\mathrm{n}\,2} + D_\mathrm{u}^{\mathrm{z}\,2}$$

$$P = \|\boldsymbol{i}_\mathrm{a}\|\,\|\boldsymbol{u}\| = G_\mathrm{e}\|\boldsymbol{u}\|^2$$

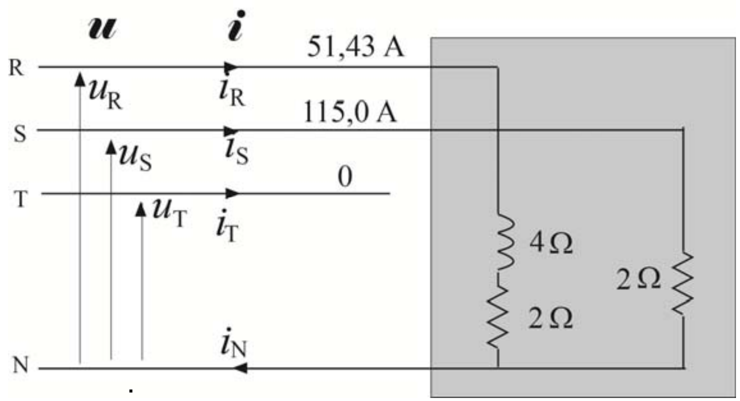
$$Q \overset{\mathrm{df}}{=} \pm\|\boldsymbol{i}_\mathrm{r}\|\,\|\boldsymbol{u}\| = -B_\mathrm{e}\|\boldsymbol{u}\|^2$$

$$D_\mathrm{u}^\mathrm{n} \overset{\mathrm{df}}{=} \|\boldsymbol{i}_\mathrm{u}^\mathrm{n}\|\,\|\boldsymbol{u}\| = A^\mathrm{n}\|\boldsymbol{u}\|^2$$

$$D_\mathrm{u}^\mathrm{z} \overset{\mathrm{df}}{=} \|\boldsymbol{i}_\mathrm{u}^\mathrm{z}\|\,\|\boldsymbol{u}\| = A^\mathrm{z}\|\boldsymbol{u}\|^2$$



U = 230V



$$Y_R = \frac{1}{2 + j4} = 0,10 - j0,20 \text{ S}, \quad Y_S = 0,50 \text{ S} \quad Y_T = 0.$$

$$||i|| = \sqrt{I_R^2 + I_S^2 + I_T^2 +} = \sqrt{51,43^2 + 115,0^2} = 126,0 \text{ A}.$$

$$||u|| = \sqrt{3}\, U = \sqrt{3} \times 230 = 398,4 \text{ V}$$

$$Y_e = G_e + jB_e = \frac{1}{3}(Y_R + Y_S + Y_T) = 0,02 - j0,067 \text{ S}.$$

$$A^n = \frac{1}{3}(Y_R + \alpha Y_S + \alpha^* Y_T) = 0,0924\, e^{122,7^0} \text{ S}$$

$$A^z = \frac{1}{3}(Y_R + \alpha^* Y_S + \alpha Y_T) = 0,217\, e^{-j103^0} \text{ S}.$$

$$P = G_e ||u||^2 = 0,20 \times (398,4)^2 = 31,7 \text{ kW}$$

$$Q = -B_e ||u||^2 = 0,0667 \times (398,4)^2 = 10,6 \text{ kvar}$$

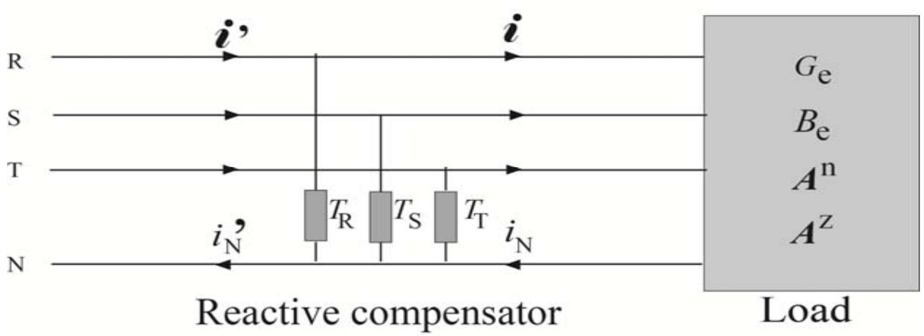
$$D_u^n = A^n ||u||^2 = 0,0924 \times (398,4)^2 = 14,7 \text{ kVA}$$

$$D_u^z = A^z ||u||^2 = 0,217 \times (398,4)^2 = 34,4 \text{ kVA}.$$

$$S = \sqrt{P^2 + Q^2 + D_u^{n2} + D_u^{z2}} = \sqrt{31,7^2 + 10,6^2 + 14,7^2 + 34,4^2} = 50,2 \text{ kVA}$$

Kompensacja reaktancyjna w obwodach trójfazowych z przewodem zerowym

$$\lambda = \frac{P}{S} = \frac{||\boldsymbol{i}_a||}{||\boldsymbol{i}||} = \frac{||\boldsymbol{i}_a||}{\sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u^n||^2 + ||\boldsymbol{i}_u^z||^2}}.$$

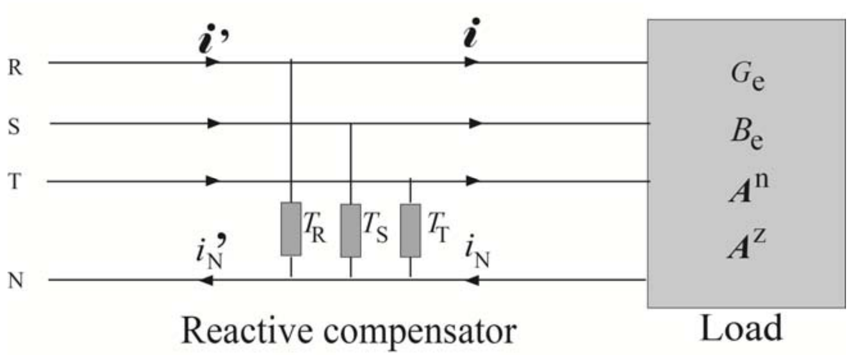


$$\boldsymbol{i}'_r \equiv 0 \quad \text{if} \quad \frac{1}{3}(T_R + T_S + T_T) + B_e = 0.$$

$$\boldsymbol{i}'^n_u \equiv 0 \quad \text{if} \quad \frac{1}{3} j(T_R + \alpha T_S + \alpha^* T_T) + \boldsymbol{A}^n = 0$$

$$\boldsymbol{i}'^z_u \equiv 0 \quad \text{if} \quad \frac{1}{3} j(T_R + \alpha^* T_S + \alpha T_T) + \boldsymbol{A}^z = 0$$

5 równań, 3 niewiadome: sprzeczny układ równań



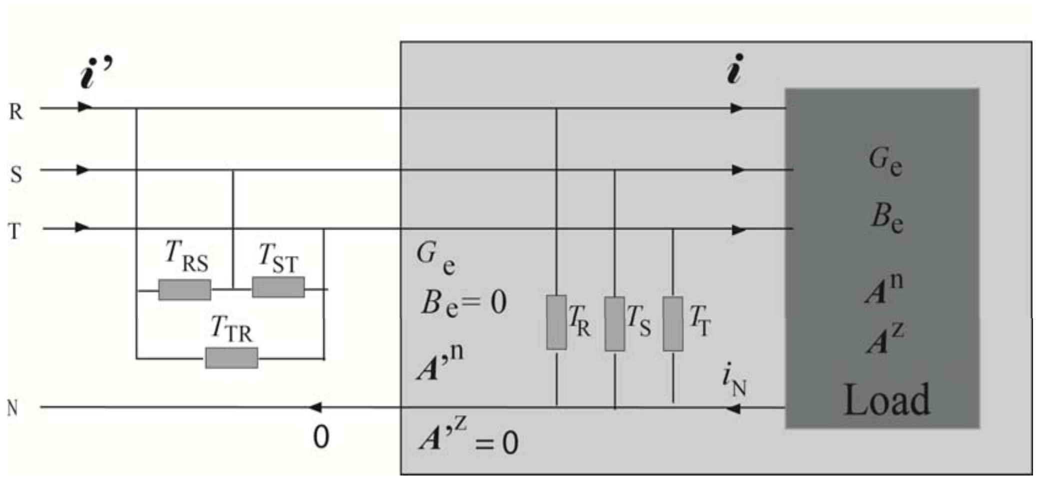
$$\boldsymbol{i}'_r \equiv 0 \quad \text{if} \quad \frac{1}{3}(T_R + T_S + T_T) + B_e = 0.$$

$$\boldsymbol{i}'^{z}_{u} \equiv 0 \quad \text{if} \quad \frac{1}{3} j\,(T_R + \alpha^* T_S + \alpha T_T) + \boldsymbol{A}^z = 0$$

$$T_R = -2\,\mathrm{Im}\,\boldsymbol{A}^z - B_e$$

$$T_S = -\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A}^z + \mathrm{Im}\,\boldsymbol{A}^z - B_e$$

$$T_T = \sqrt{3}\,\mathrm{Re}\,\boldsymbol{A}^z + \mathrm{Im}\,\boldsymbol{A}^z - B_e\,.$$



$$\boldsymbol{i}'^{n}_{u} \equiv 0 \quad \text{if} \quad j\,(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS}) + \boldsymbol{A}'^{n} = 0$$

$$T_{RS} = (\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A}_u'^{n} - \mathrm{Im}\,\boldsymbol{A}_u'^{n})/3$$

$$T_{ST} = (2\,\mathrm{Im}\,\boldsymbol{A}_u'^{n})/3$$

$$T_{TR} = (-\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A}_u'^{n} - \mathrm{Im}\,\boldsymbol{A}_u'^{n})/3$$

Kompensator Y składowej zerowej:

$$T_{\mathrm{R}} = -2\,\mathrm{Im}\boldsymbol{A}^{\mathrm{z}} - B_{\mathrm{e}} = -\,0.289 \;\; \mathrm{S}$$

$$T_{\mathrm{S}} = -\sqrt{3}\,\mathrm{Re}\boldsymbol{A}^{\mathrm{z}} + \mathrm{Im}\boldsymbol{A}^{\mathrm{z}} - B_{\mathrm{e}} = 0.289 \;\; \mathrm{S}$$

$$T_{\mathrm{T}} = \sqrt{3}\,\mathrm{Re}\boldsymbol{A}^{\mathrm{z}} + \mathrm{Im}\boldsymbol{A}^{\mathrm{z}} - B_{\mathrm{e}} = 0.50 \;\; \mathrm{S}.$$

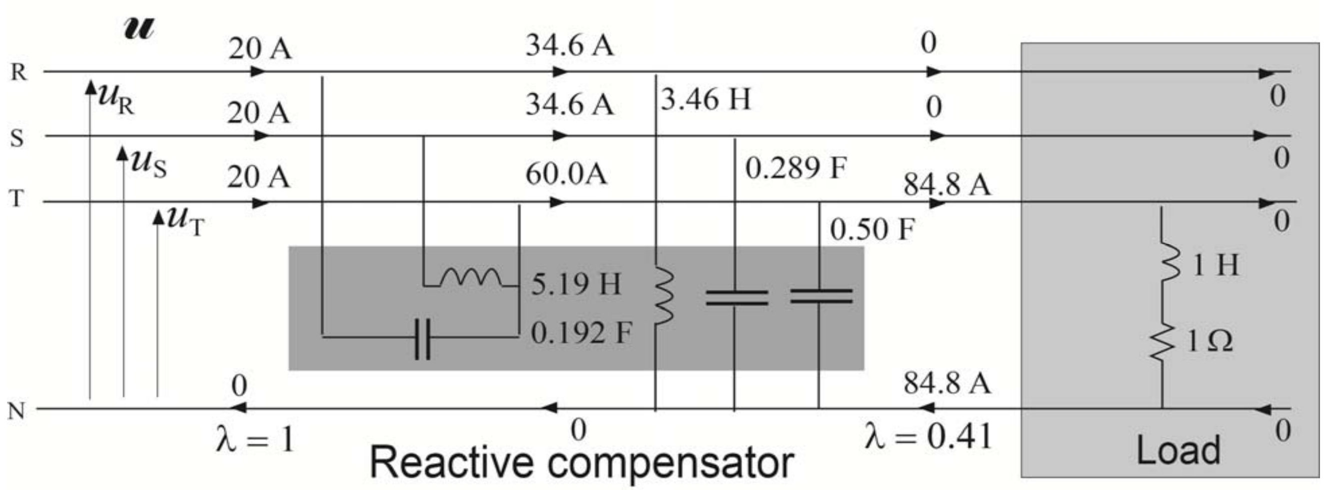
$$\boldsymbol{A}'^{\,\mathrm{n}} = \boldsymbol{A}^{\mathrm{z}*} + \boldsymbol{A}^{\mathrm{n}} = (0.061 + j\,0.228)* - 0.228 - j\,0.061 = -0.167 - j\,0.289 \;\; \mathrm{S}$$

Kompensator Δ składowej ujemnej:

$$T_{\mathrm{RS}} = (\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A}'^{\,\mathrm{n}} - \mathrm{Im}\,\boldsymbol{A}'^{\,\mathrm{n}})/3 = 0$$

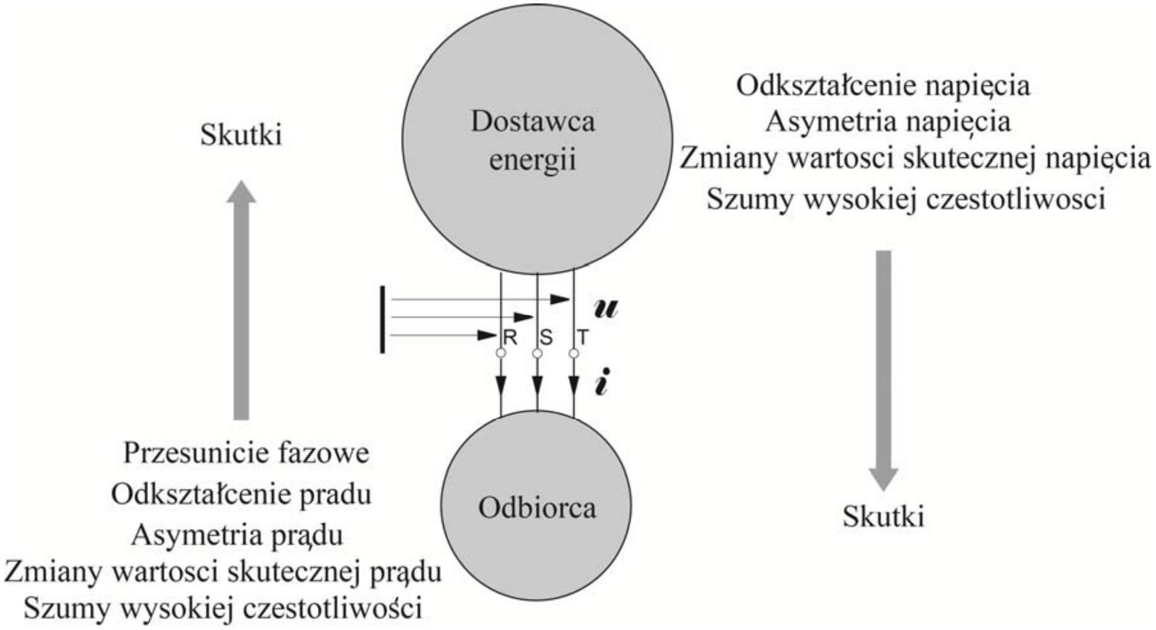
$$T_{\mathrm{ST}} = (2\,\mathrm{Im}\,\boldsymbol{A}'^{\,\mathrm{n}})/3 = -0.192 \;\mathrm{S}$$

$$T_{\mathrm{TR}} = (-\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A}'^{\,\mathrm{n}} - \mathrm{Im}\,\boldsymbol{A}'^{\,\mathrm{n}})/3 = 0.192 \;\mathrm{S}.$$



Moce i Kompensacja
w Obwodach
z Niesinusoidalnymi Przebiegami
Prądu i Napięcia

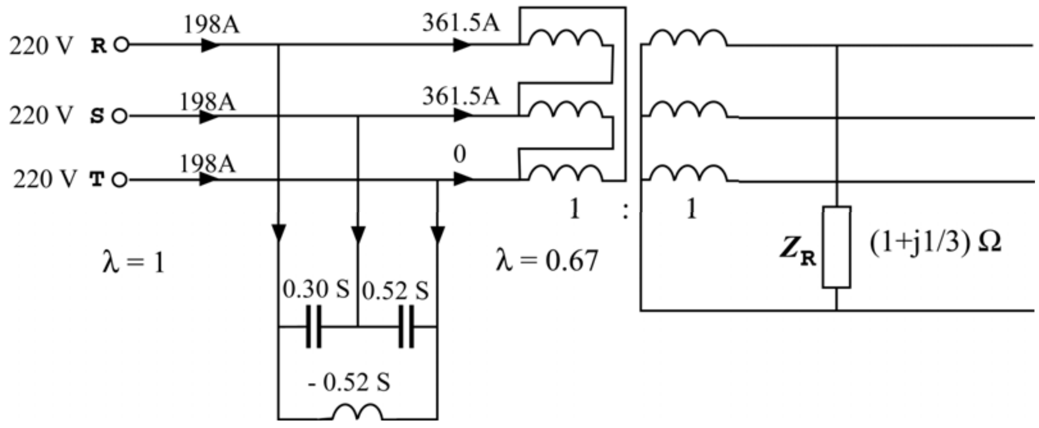
Cz. 3. Kompensacja



Tradycyjny cel kompensacji to kompensacja mocy biernej (przesunięcia fazowego prądu)
(kompensatory pojemnościowe, maszyny synchroniczne)

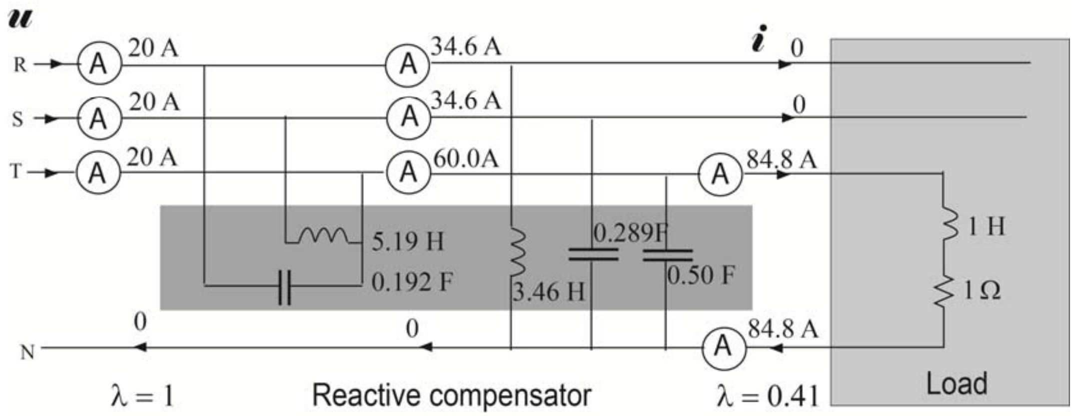
oraz
filtracja harmonicznych
(rezonansowe filtry harmonicznych, filtry pasmowe)

Reaktancyjny kompensator równoważący w obwodzie trójprzewodowym



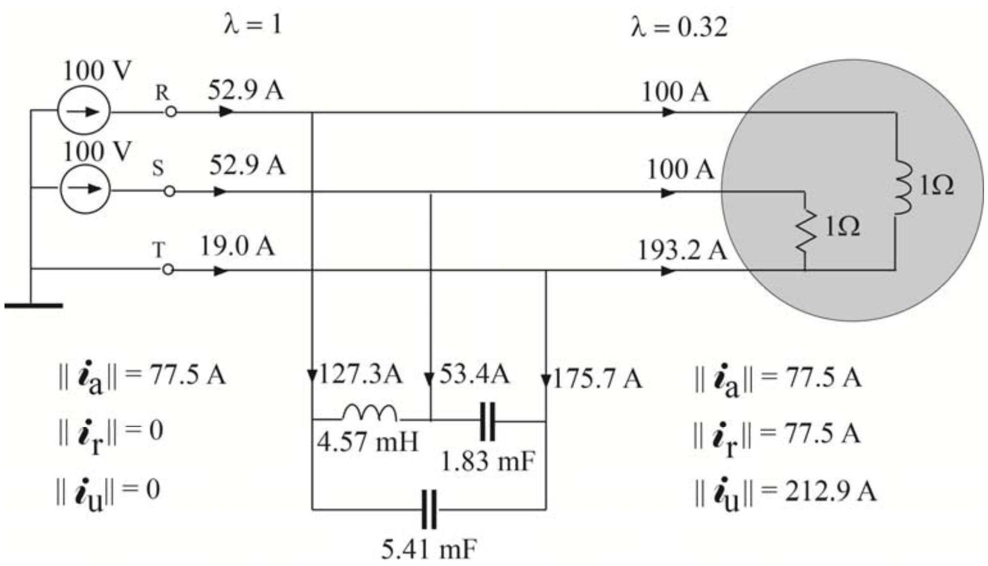
$\|\boldsymbol{i}_{\mathrm{a}}\| = 343$ A, $\|\boldsymbol{i}_{\mathrm{u}}\| = 0$, $\|\boldsymbol{i}_{\mathrm{r}}\| = 0$, $\|\boldsymbol{i}\| = 343$ A, $S = 131$ kVA, $\lambda = 1$

Reaktancyjny kompensator równoważący
w obwodzie czteroprzewodowym



L.S. Czarnecki, P. M. Haley, "Unbalanced Power in Four-Wire Systems and its Reactive Compensation",
w druku w IEEE Trans. on Power Delivery, 2014.

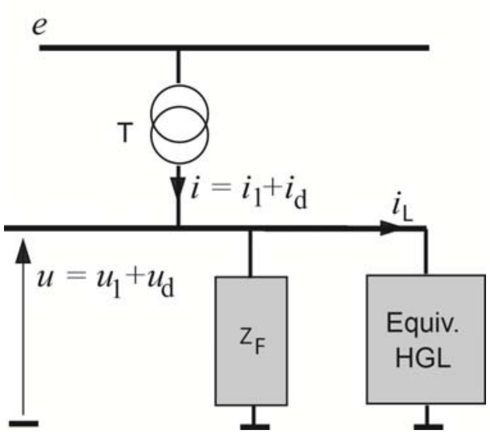
Równoważąca kompensacja reaktancyjna w warunkach asymetrii napięciowej

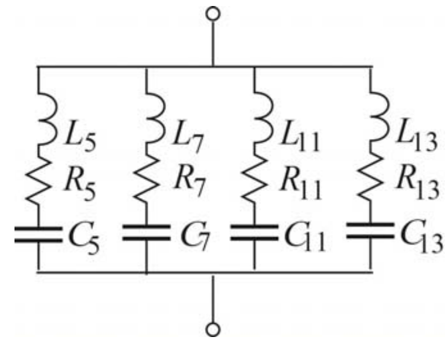


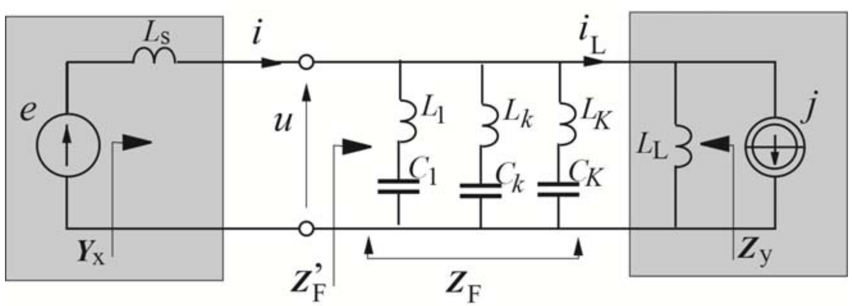
$$\boldsymbol{i} = G_b(\boldsymbol{u}^p + \boldsymbol{u}^n), \qquad G_b = \frac{P}{||\boldsymbol{u}^p||^2 + ||\boldsymbol{u}^n||^2}$$

L.S. Czarnecki, P. Bhattarai, ”Powers and Reactive Compensation of Unbalanced Loads with Asymmetrical Voltages”,
IEEE Transactions on Power Delivery (w recenzji)

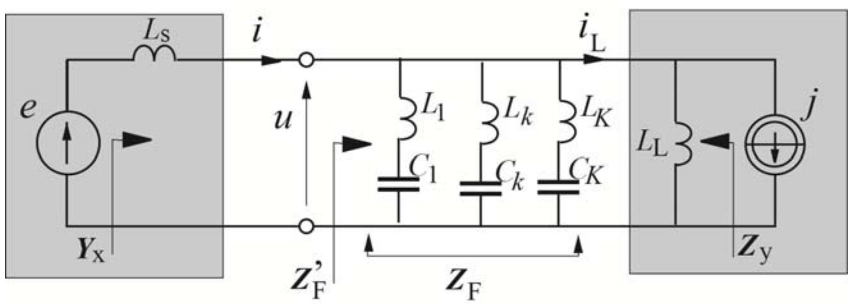
Rezonansowe filtry harmonicznych





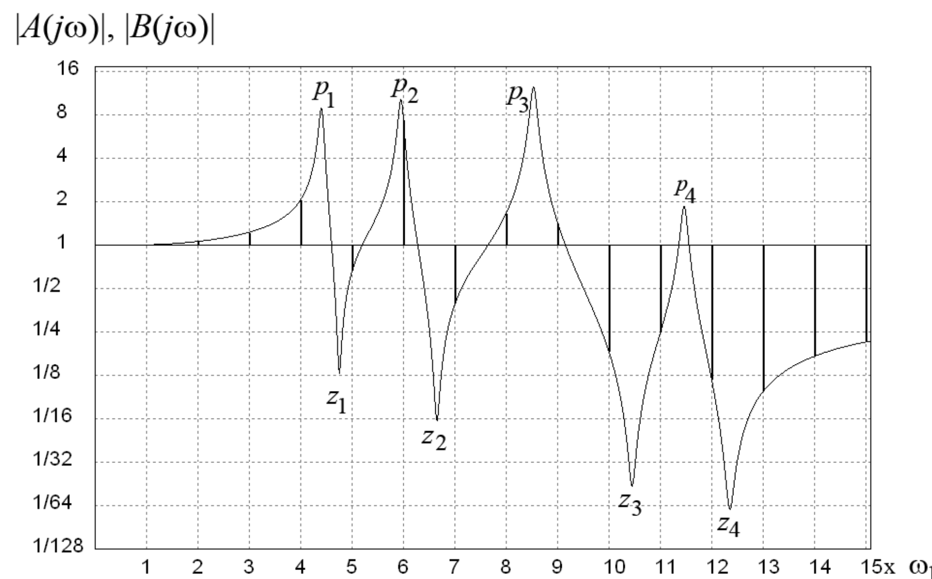


Rezonansowe filtry harmonicznych

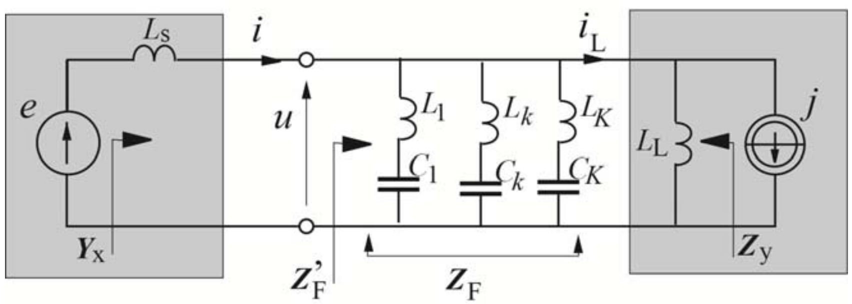


$$A(j\omega) \overset{\text{df}}{=} \left. \frac{U(j\omega)}{E(j\omega)} \right|_{j(t) \equiv 0} = \frac{Z'_F(j\omega)}{Z_s(j\omega) + Z'_F(j\omega)}$$

$$B(j\omega) \overset{\text{df}}{=} \left. \frac{I(j\omega)}{J(j\omega)} \right|_{e(t) \equiv 0} = \frac{Z'_F(j\omega)}{Z_s(j\omega) + Z'_F(j\omega)}$$

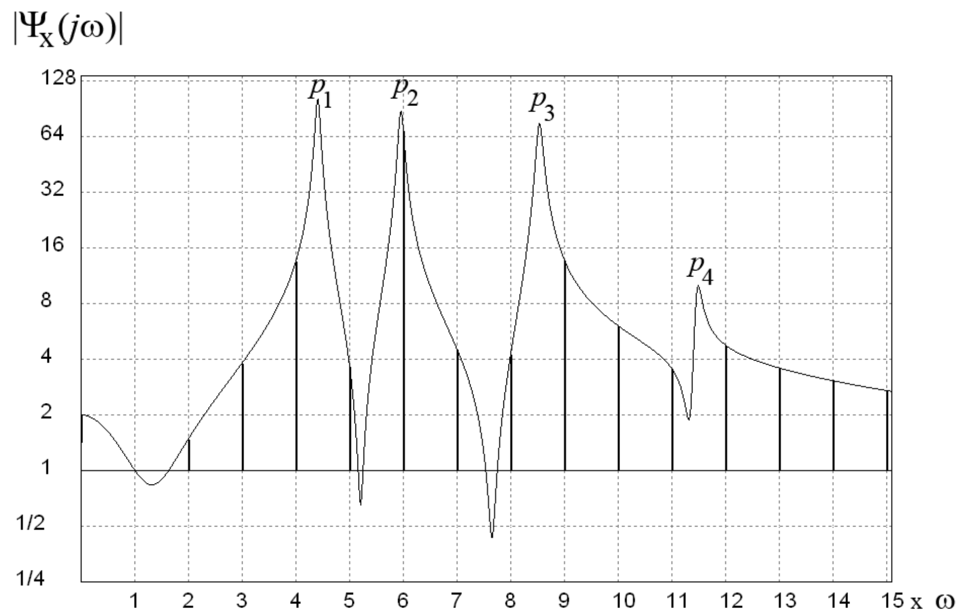


Rezonansowe filtry harmonicznych



$$Y_x(j\omega) \overset{\mathrm{df}}{=} \left.\frac{I(j\omega)}{E(j\omega)}\right|_{j(t)\equiv 0} = \frac{1}{Z_s(j\omega) + Z'_F(j\omega)}$$

$$\Psi_x(j\omega) = \frac{Y_x(j\omega)}{Y_x(j\omega_1)}$$



Skuteczność filtru:

$$\varepsilon_i \stackrel{df}{=} 1 - \frac{\delta_i}{\delta_{i0}} \qquad \varepsilon_u \stackrel{df}{=} 1 - \frac{\delta_u}{\delta_{u0}}$$

Prostownik trójfazowy

Efficiency of Type A and Type B filters at internal voltage distortion **δ_e = 2.5%** and **1%** of the current distortion by non-characteristic harmonics

Filter	S_{sc}/P	20	25	30	35	40	45
A	ε_u	0.46	0.51	0.37	0.16	0.16	**- 0.32**
	ε_i	0.11	0.33	0.23	**- 0.10**	**0.02**	**- 0.47**
B	ε_u	**- 0.26**	0.20	0.36	0.37	0.33	0.32
	ε_i	**- 0.50**	**- 0.03**	0.29	0.36	0.36	0.34

L.S. Czarnecki and H.L. Ginn, “The effect of the design method on efficiency of resonant harmonic filters“ *IEEE Trans. on Power Delivery*, Vol. 20, No. 1, pp. 286-291, 2005.

Efficiency of optimized filter at internal voltage distortion δ_e **= 2.5%** and **1%** of the current distortion by non-characteristic harmonics

$$\delta = W_c\,\delta_i + W_v\,\delta_u$$

$$\delta = f(a_5,\ldots a_{13},\, t_5,\ldots t_{13})$$

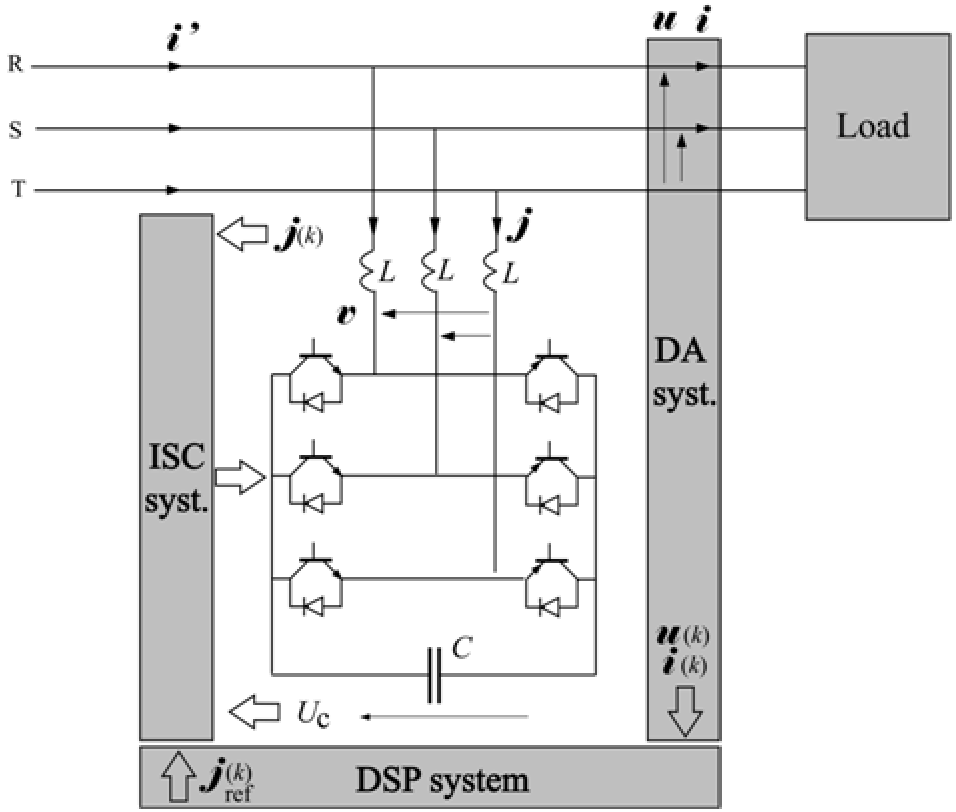
a_n – współczynnik alokacji mocy biernej do filtru n-tej harmonicznej

t_n – częstotliwość strojenia filtru n-tej harmonicznej

S_{sc}/P	-	20	25	30	35	40	45
ε_u	%	0.67	0.62	0.58	0.53	0.49	0.44
ε_i	%	0.57	0.54	0.52	0.48	0.44	0.39

Równoległy kompensator kluczujący (Switching compensator)

Znany też pod błędnymi nazwami, jako
"Active power filter", "Active harmonic filter", "Power conditioner"



Teoria Chwilowej Mocy Biernej p-q

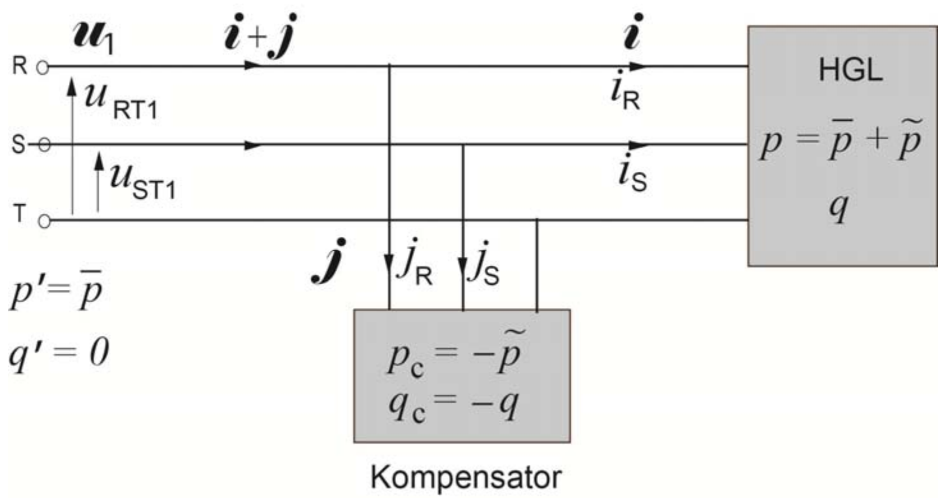
Przekształcenie Clarke'a w układzie trójprzewodowym:

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_S \end{bmatrix} \qquad \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \end{bmatrix}$$

$$p = u_\alpha i_\alpha + u_\beta i_\beta$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha$$

$$p = \overline{p} + \tilde{p}$$

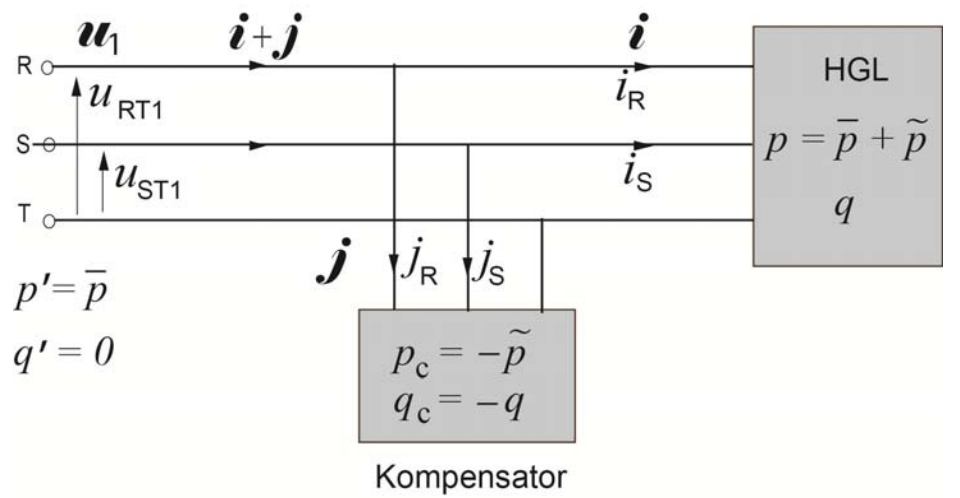


$$p = u_\alpha i_\alpha + u_\beta i_\beta$$

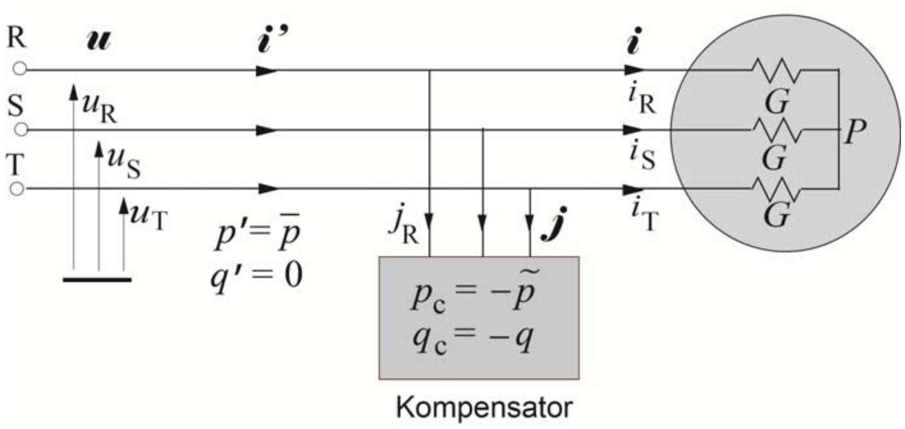
$$q = u_\alpha i_\beta - u_\beta i_\alpha$$

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} u_\alpha , & u_\beta \\ -u_\beta , & u_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathsf{U}_\mathrm{C} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

$$\boldsymbol{j} \overset{\mathrm{df}}{=} \begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \frac{1}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha , & -u_\beta \\ u_\beta , & u_\alpha \end{bmatrix} \begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} = \mathsf{U}_\mathrm{C}^{-1} \begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix}$$



L.S. Czarnecki, (2009), “Effect of supply voltage harmonics on IRP-based switching compensator control”, *IEEE Trans. on Power Electronics*, Vol. 24, No. 2, pp. 483-488.



$$u_{R1} = \sqrt{2}\, U_1 \cos \omega_1 t\,, \qquad u_{R5} = \sqrt{2}\, U_5 \cos 5\omega_1 t$$

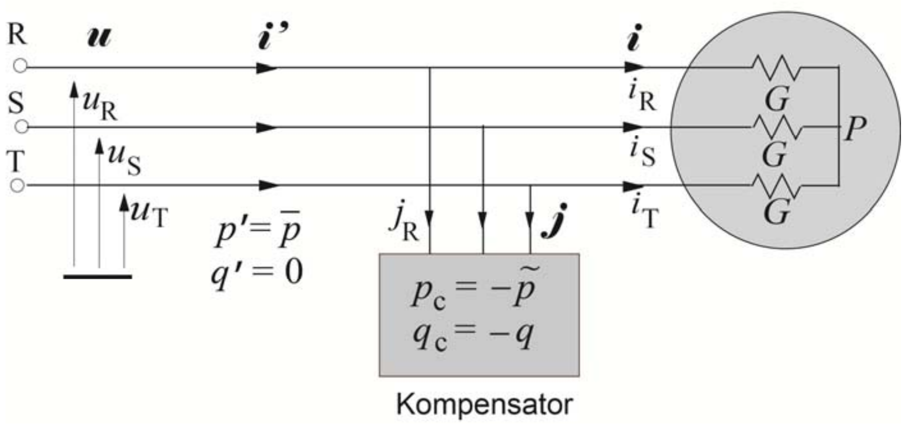
Algorytm sterowania oparty na Teorii CMB p-q powoduje wytwarzanie
przez kompensator prądu w przewodzie R

$$j_R = \frac{-2\sqrt{2}\, G U_1 U_5 \cos 6\omega_1 t}{U_1^2 + U_5^2 + 2U_1 U_5 \cos 6\omega_1 t}(U_1 \cos \omega_1 t + U_5 \cos 5\omega_1 t)$$

Jest tak dlatego, że odkształcenie napięcia zasilania powoduje
oscylacje chwilowej mocy czynnej. W szczególności, przy 5-tej harmonicznej:

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \overline{p} + \tilde{p} = 3G(U_1^2 + U_5^2) + 6GU_1U_5 \cos 6\omega_1 t$$

L.S. Czarnecki, (2010), ”Effect of supply voltage asymmetry on IRP p-q - based switching compensator control”, *IET Proc. on Power Electronics*, Vol. 3, No. 1, pp. 11-17



$$u_R^p = \sqrt{2}\, U^p \cos\omega_1 t, \qquad u_R^n = \sqrt{2}\, U^n \cos\omega_1 t$$

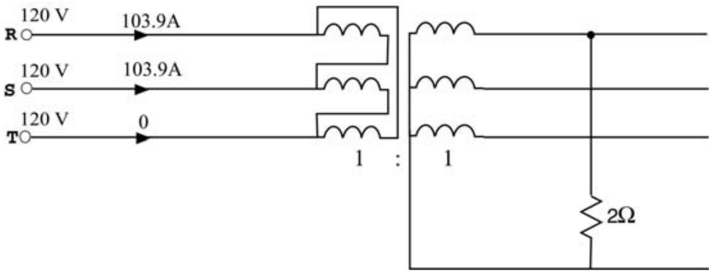
Algorytm sterowania oparty na Teorii CMB p-q powoduje wytwarzanie przez kompensator prądu w przewodzie R

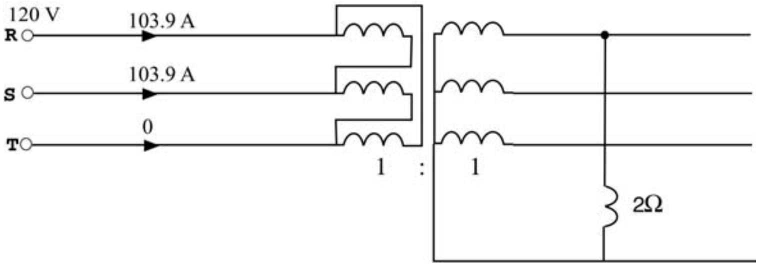
$$j_R = \frac{-2\sqrt{2}\, G(U^p + U^n) U^p U^n}{U^{p2} + U^{n2} + 2U^p U^n \cos 2\omega_1 t} \cos\omega_1 t \cos 2\omega_1 t$$

Jest tak dlatego, że asymetria napięcia zasilania powoduje oscylacje chwilowej mocy czynnej:

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \bar{p} + \tilde{p} = 3G(U^{p2} + U^{n2}) + 6GU^p U^n \cos 2\omega_1 t$$

L.S. Czarnecki, "On some misinterpretations of the Instantaneous Reactive Power p-q Theory", IEEE Trans. on Power Electronics, Vol. 19, No.3, pp. 828-836, 2004





$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}UI[1 + \cos 2(\omega_1 t + 30^0)]$$

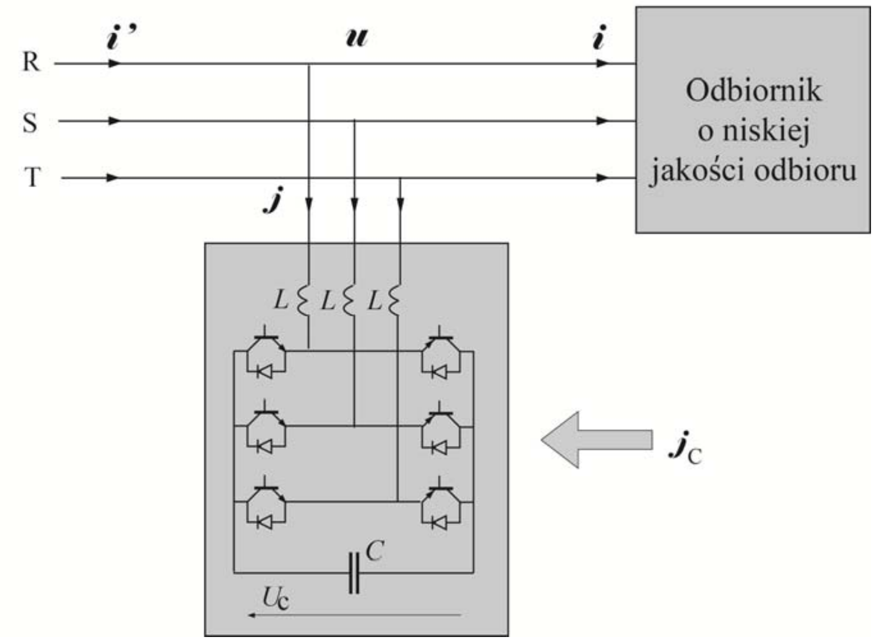
$$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}UI\sin 2(\omega_1 t + 30^0)$$

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}UI\cos(2\omega_1 t - 30^0)$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}UI[1 + \sin(2\omega_1 t - 30^0)]$$

Istnieją chwile czasu, w których moce *p* i *q*
w obu obwodach są identyczne

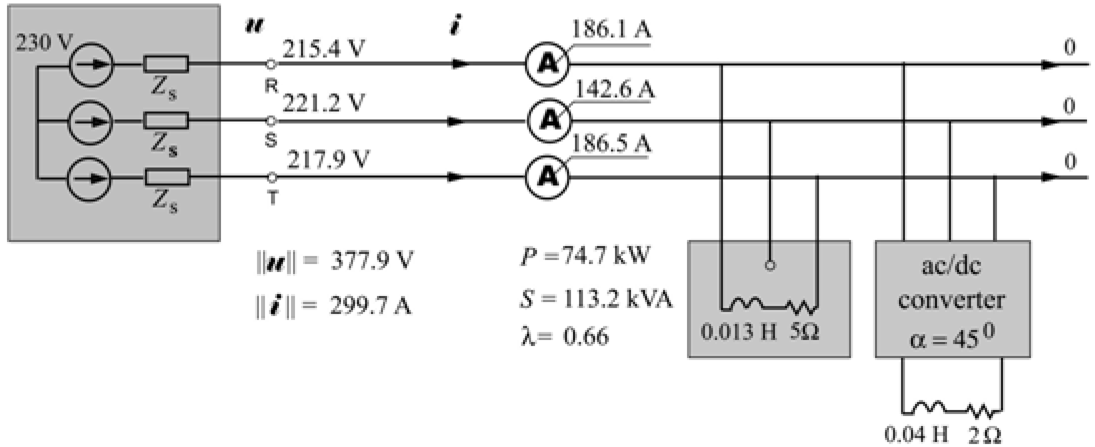
Cele kompensacji równoległej

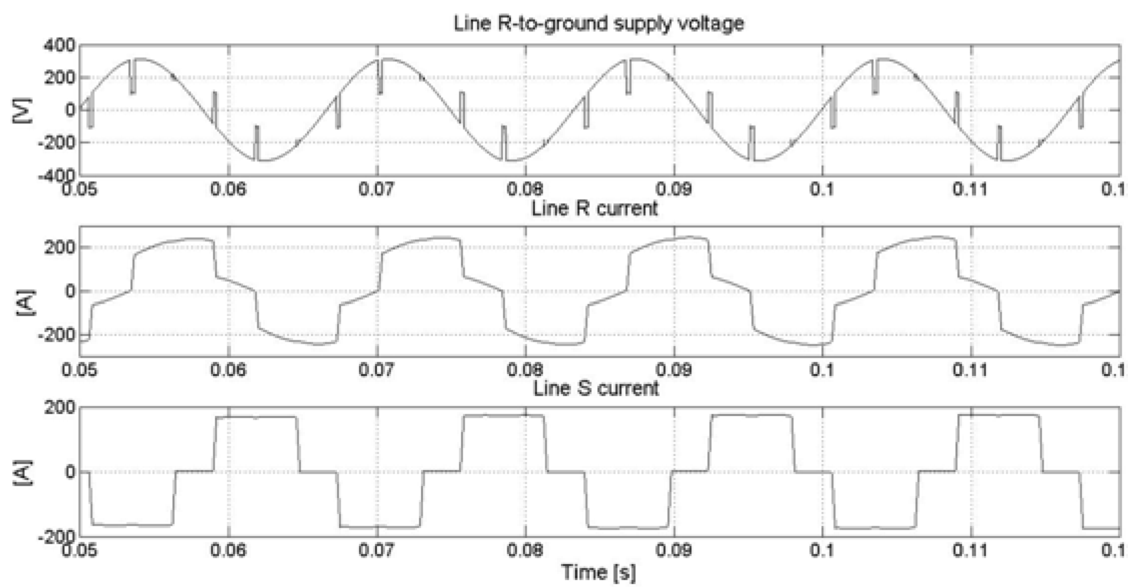


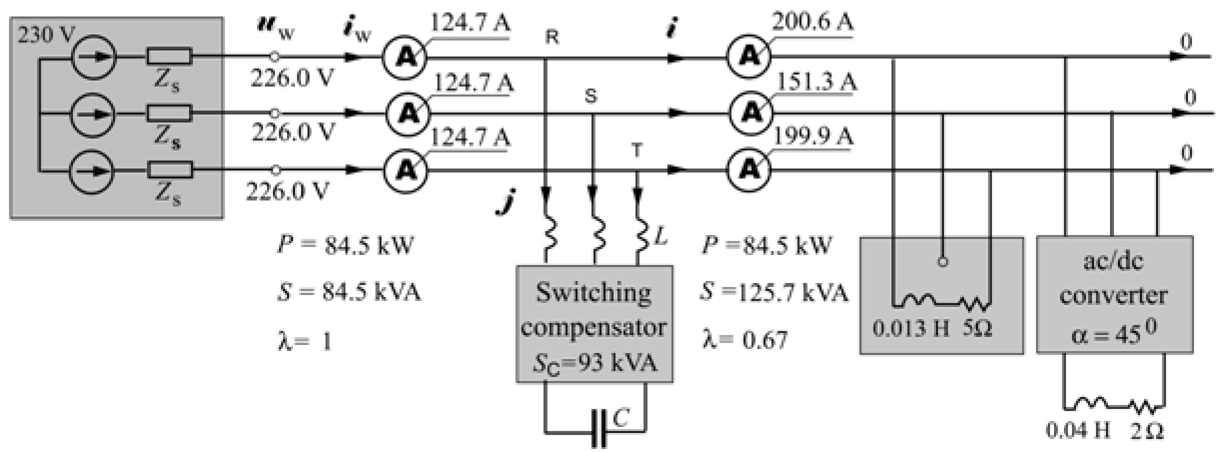
$$\boldsymbol{i}' = \boldsymbol{i}_{aF}(t), \qquad \boldsymbol{i}_{aF}(t) \stackrel{df}{=} \frac{P}{\|\boldsymbol{u}\|^2}\boldsymbol{u}(t), \qquad \boldsymbol{j}_c = -(\boldsymbol{i} - \boldsymbol{i}_{aF})$$

$$\boldsymbol{i}' = \boldsymbol{i}_{aC}(t), \qquad \boldsymbol{i}_{aC}(t) \stackrel{df}{=} \frac{P_C}{\|\boldsymbol{u}_C\|^2}\boldsymbol{u}_C(t), \qquad \boldsymbol{j}_c = -(\boldsymbol{i} - \boldsymbol{i}_{aC})$$

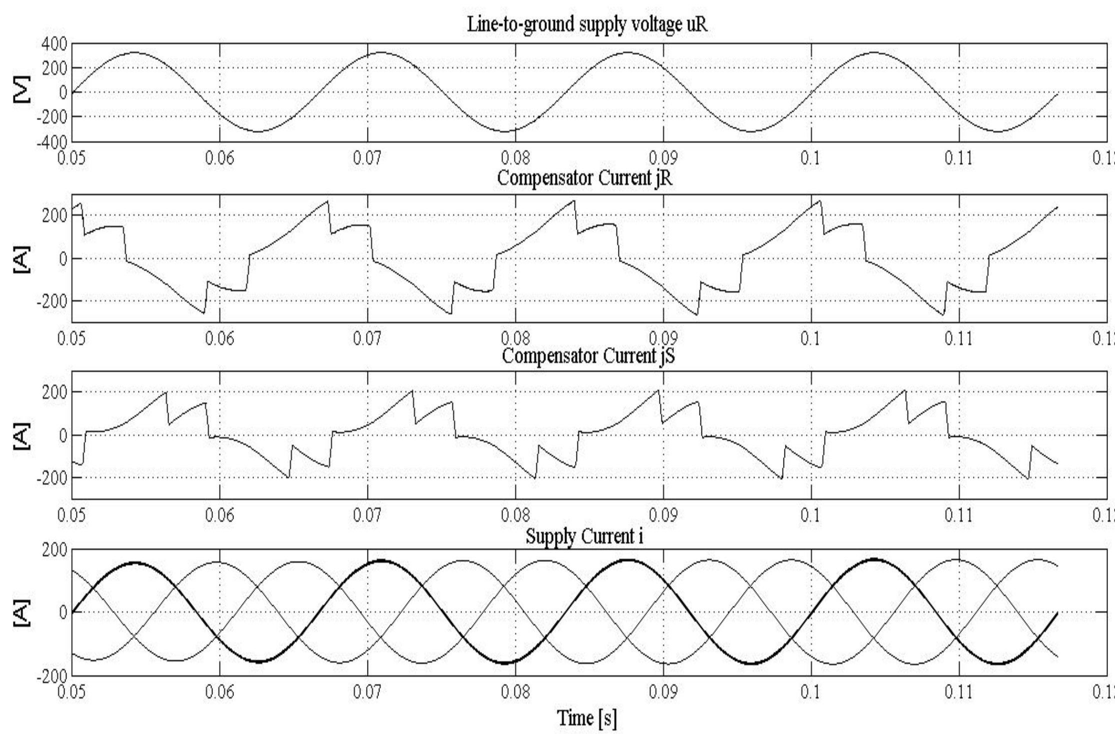
$$\boldsymbol{i}' = \boldsymbol{i}_{w}(t), \qquad \boldsymbol{i}_{w}(t) \stackrel{df}{=} \frac{P_w}{\|\boldsymbol{u}_1^p\|^2}\boldsymbol{u}_1^p(t), \qquad \boldsymbol{j}_c = -(\boldsymbol{i} - \boldsymbol{i}_{w})$$


230 V
u
i
215.4 V
221.2 V
217.9 V
R
S
T
Z_s
186.1 A
142.6 A
186.5 A
0
0
0
‖u‖ = 377.9 V
‖i‖ = 299.7 A
P = 74.7 kW
S = 113.2 kVA
λ = 0.66
0.013 H 5Ω
ac/dc
converter
α = 45 0
0.04 H 2 Ω

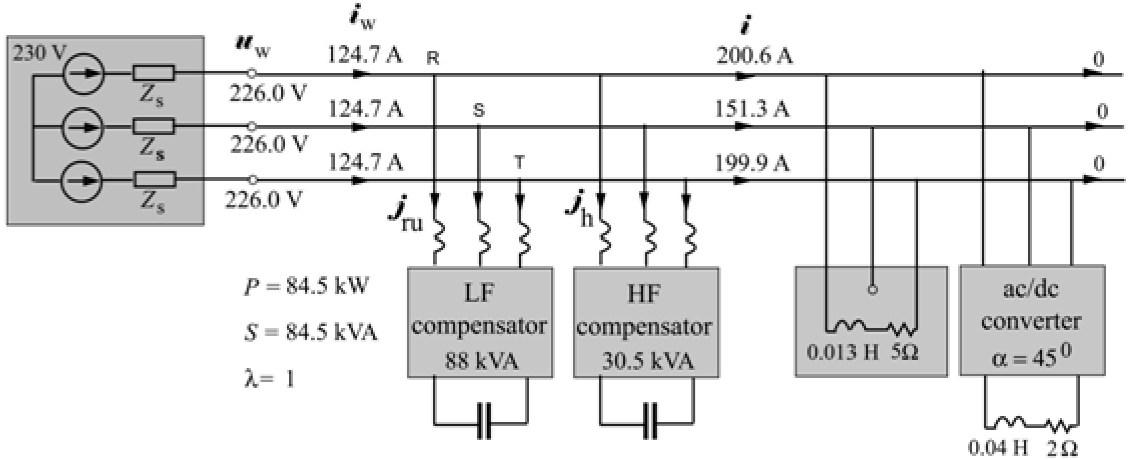

Line R-to-ground supply voltage
[V]
Line R current
[A]
Line S current
[A]
Time [s]

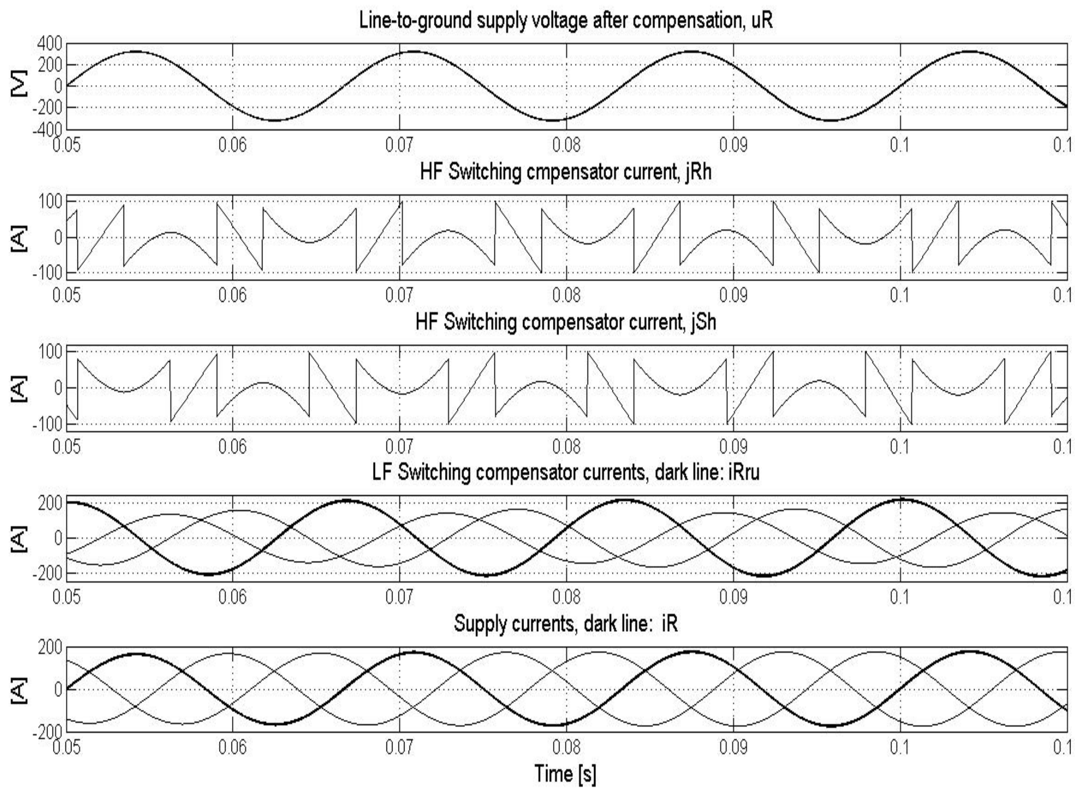


230 V
Z_s
u_w
i_w
226.0 V
124.7 A
R
S
T
i
200.6 A
151.3 A
199.9 A
0
j
L
P = 84.5 kW
S = 84.5 kVA
λ= 1
Switching compensator
S_C=93 kVA
C
P =84.5 kW
S =125.7 kVA
λ= 0.67
0.013 H 5Ω
ac/dc converter
α = 45⁰
0.04 H 2Ω

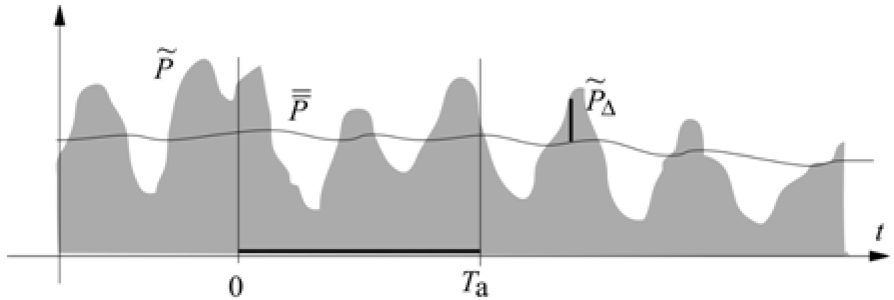


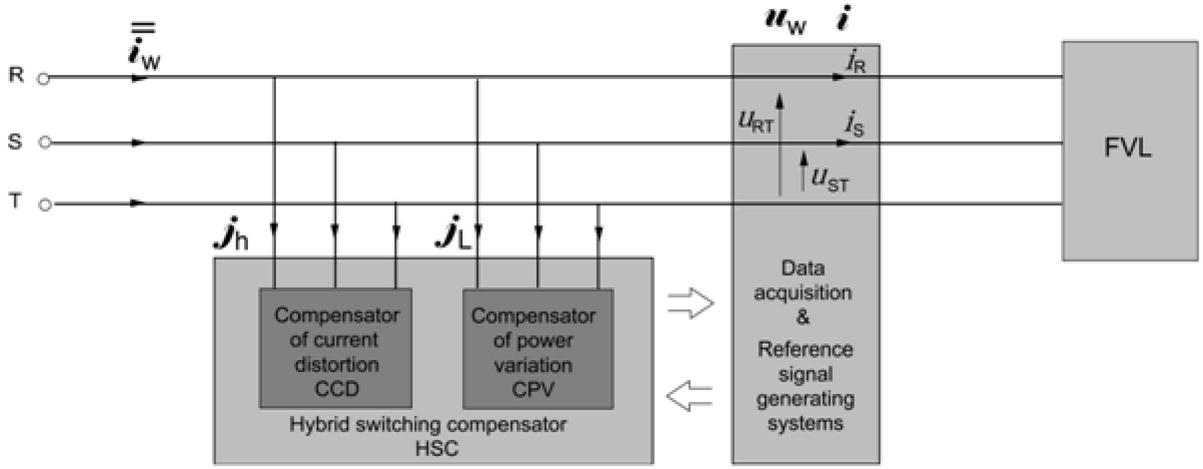
Line-to-ground supply voltage uR
Compensator Current jR
Compensator Current jS
Supply Current i
[V]
[A]
Time [s]

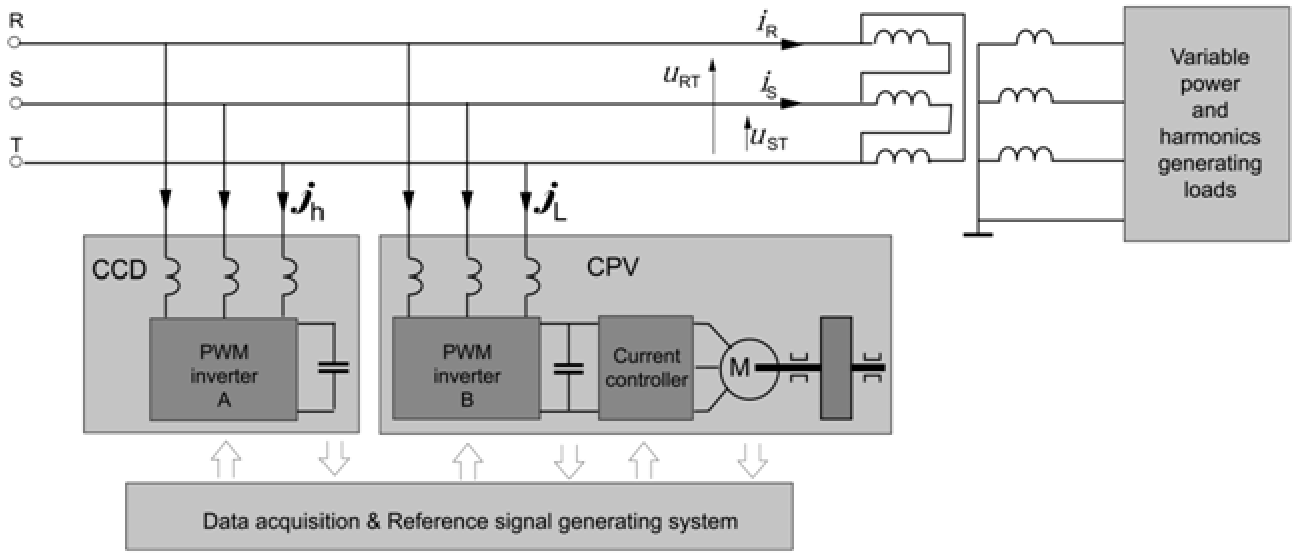

230 V
Z_s
u_w
226.0 V
226.0 V
226.0 V
i_w
124.7 A
124.7 A
124.7 A
R
S
T
j_{ru}
j_h
i
200.6 A
151.3 A
199.9 A
0
0
0
P = 84.5 kW
S = 84.5 kVA
λ= 1
LF compensator 88 kVA
HF compensator 30.5 kVA
0.013 H 5Ω
ac/dc converter α = 45⁰
0.04 H 2Ω

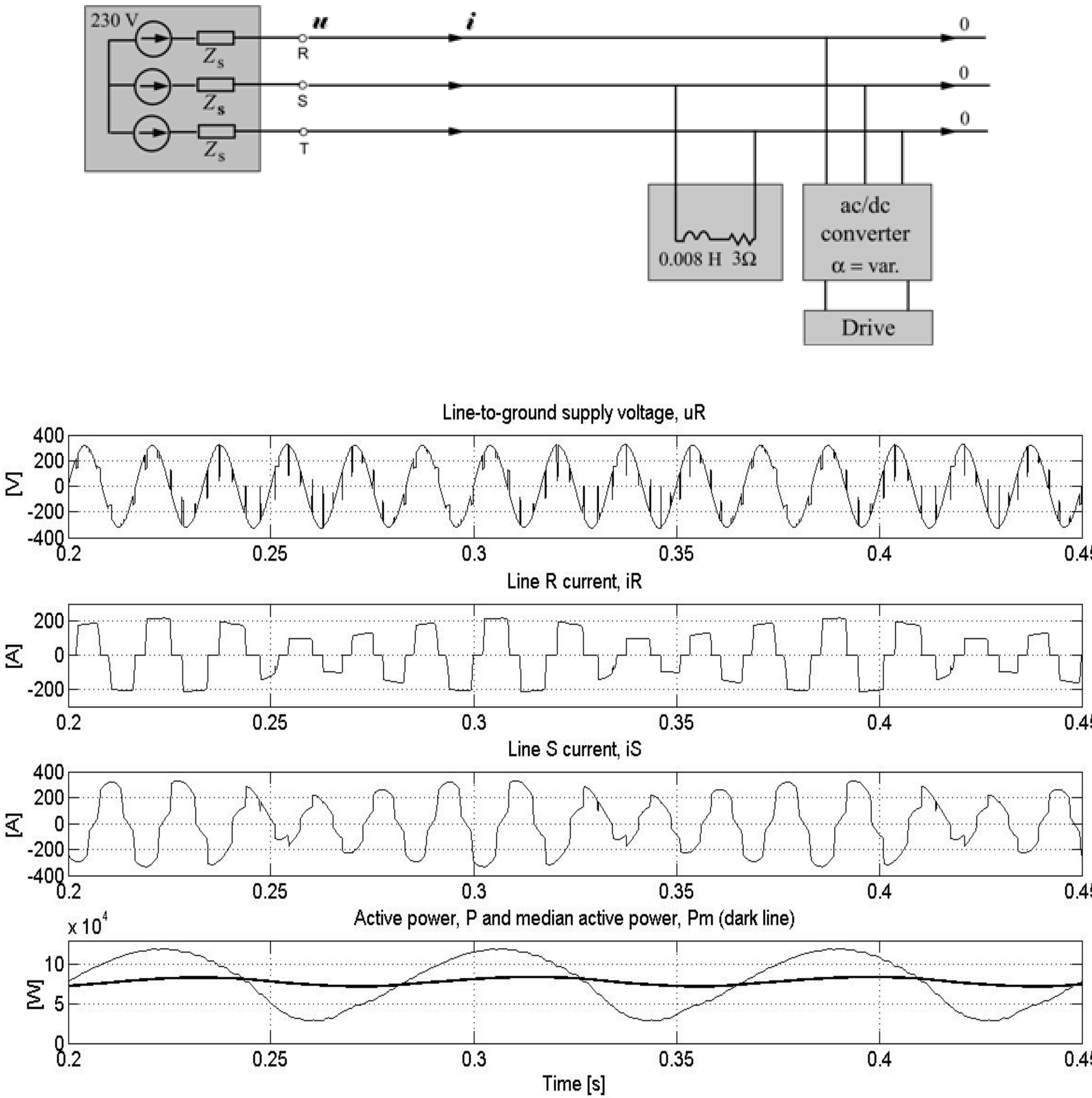

Line-to-ground supply voltage after compensation, uR
HF Switching cmpensator current, jRh
HF Switching compensator current, jSh
LF Switching compensator currents, dark line: iRru
Supply currents, dark line: iR
[V]
[A]
Time [s]

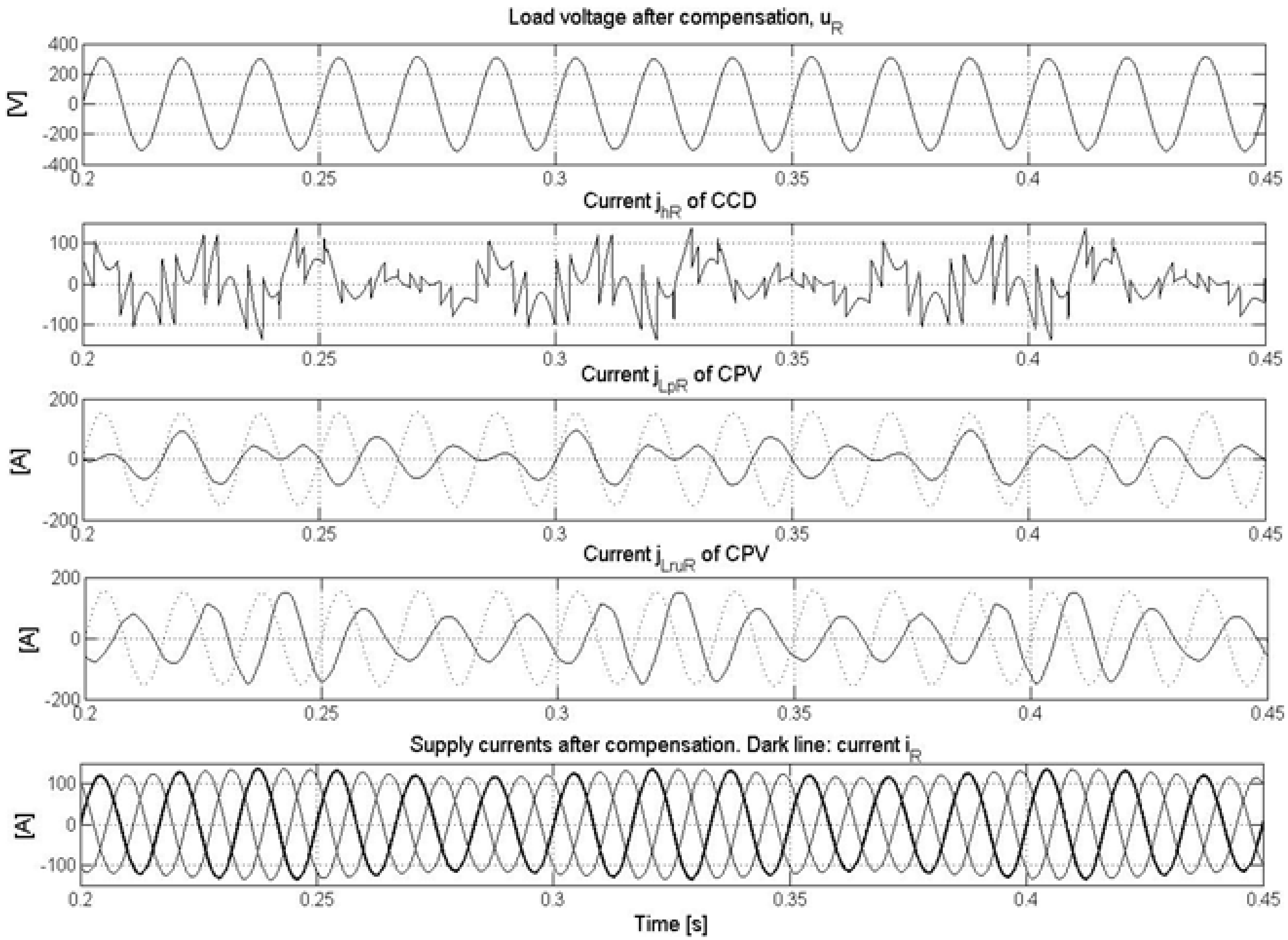
Kompensacja odbiorników o zmiennej mocy czynnej






R
S
T
i_R
u_{RT}
i_S
u_{ST}
j_h
j_L
Variable power and harmonics generating loads
CCD
PWM inverter A
CPV
PWM inverter B
Current controller
M
Data acquisition & Reference signal generating system


230 V
Zs
R
S
T
u
i
0
0.008 H 3Ω
ac/dc
converter
α = var.
Drive
Line-to-ground supply voltage, uR
[V]
Line R current, iR
[A]
Line S current, iS
[A]
Active power, P and median active power, Pm (dark line)
x 10^4
[W]
Time [s]


Load voltage after compensation, u_R
[V]
400
200
0
-200
-400
0.2
0.25
0.3
0.35
0.4
0.45
Current j_hR of CCD
100
0
-100
Current j_LpR of CPV
[A]
200
0
-200
Current j_LruR of CPV
[A]
Supply currents after compensation. Dark line: current i_R
[A]
Time [s]

Dziękuję za uwagę !

Strona internetowa z pewną liczbą artykułów w formacie PDF:
www.lsczar.info

Adres e_mailowy:
lsczar@cox.net

L.S. Czarnecki, (2005), Moce w Obwodach Elektrycznych z Niesinusoidalnymi Przebiegami Prądów i Napięć, *Oficyna Wydawnicza Politechniki Warszawskiej*