Keynote presentation on 2016 International Conference on Applied and Theoretical Electr. Craiova, Romania

From Steinmetz to Currents' Physical Components (CPC): History of Power Theory Development

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***Abstract*—The paper presents a short draft of history of the development of the power theory of systems with nonsinusoidal voltages and currents. It starts from the Steinmetz experiment run in 1892 and ends with the development of the Currents' Physical Components (CPC) – based power theory. A draft of the CPC and the major contributions to the power theory and methods of compensation are presented as well. The time-domain and frequency-domain as well as the instantaneous and the averaging approaches to the power theory and compensation are compared.**

Keywords— power definitions, compensation, unbalanced loads, harmonics, asymmetry, Conservative Power Theory, CPT, Instantaneous Reactive Power p-q Theory.

I. Introduction

The concept of power quantities for single-phase systems with sinusoidal voltages and currents, such as the active, reactive and apparent powers, P, Q and S, was developed by the end of the nineteenth century. These powers in circuits with linear, time invariant (LTI) loads satisfy the power equation

$$S^2 = P^2 + Q^2 .$$

In 1892 Steinmetz ran an experiment [1] to verify this equation, but with LTI load replaced by an arc bulb, as shown in Fig. 1.

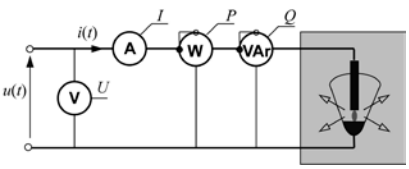

Fig. 1. Steinmetz' experiment

This experiment has demonstrated that in spite of zero reactive power Q, the apparent power, $S = U\ I$, can be higher than the load active power P, thus

$$S^2 \geq P^2 + Q^2 .$$

Since the cost of the energy delivery by its provider is associated with the apparent power S, while the value of that energy for the customer is associated with the active power P, the inequality

$$S \geq P$$

is crucially important for the power system economy.

This inequality raises two main questions. The first one is cognitive, namely

- Why can the apparent power S be higher than the active power P? What phenomena in the load are responsible for this inequality?

The second question is related to the power system economy, and it is practical in nature, namely

- How can the difference between the apparent power S and the active power P be reduced by a compensator?

The answer to these questions is the subject of ***power theory***.

The notion of "power theory" was originally introduced to the electrical engineering community in 1931 by Fryze [9] and used in phrases such as "Budeanu's power theory" or "Fryze's power theory", i.e., it was associated with specific definitions of powers and descriptions of power properties of a load. It means that we have a number of different power theories of the same electrical system. Thus, when we use the term "power theory" we have to specify which one of them we have in mind. Studies on these theories can be regarded as a subject of ***meta-theory*** [59] of electric powers.

We can use the term "power theory", as suggested in [36], in a different sense, however. The power theory can be regarded as a set of true statements on power properties of electrical loads. Any statement on power properties of such loads, as long as it is not challenged as incorrect, can be regarded as an element of the power theory. It is not personalized. Whoever reveals a new power property of electrical loads contributes to this theory development.

The arc bulb used in the Steinmetz experiment was probably of the active power of no more than a few hundred watts. AC arc furnaces used presently in metallurgy can have apparent power of the order of 750 MVA. Its supply structure is shown in Fig. 2. It is a present-time version of the Steinmetz experiment.

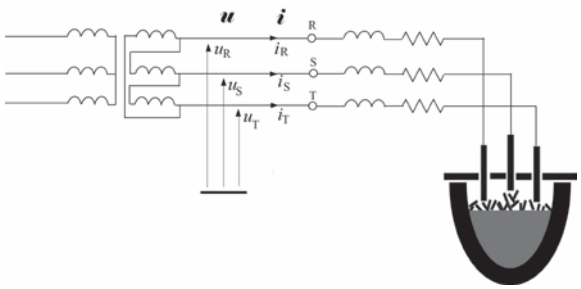

Fig. 2. AC arc furnace supply structure

This is a single device of the power comparable to a one million population city. The line current could be of the order of 600 kA, and the annual bill for the energy could be at the level of 500 million dollars. We should be aware, however, that even if the power of such a furnace is comparable to the power of a one million population city, the voltages and currents at the supply of such a city are

almost sinusoidal and symmetrical with a power factor which could be close to unity. In the case of the arc furnace, inductors inserted in the supply lines for the arc stabilization, reduce the power factor to the level of 0.7. When the arc is not ignited, one of the line currents disappears, even for such a short interval as half of the period. Strong asymmetry of the supply current and distortion suddenly occurs. The second harmonic could be of the order of one third of the fundamental harmonic. The power factor can drop even to 0.4.

II. Schools of Power Theory

Apparently easy questions inspired by the Steinmetz observation have occurred to be some of the most difficult questions of electrical engineering. Hundreds of scientists have attempted to explain and describe power properties of loads with nonsinusoidal voltages and currents and develop methods of compensation. Hundreds of papers were published. A number of "schools" of power theory were established. The most known were power theories suggested by Budeanu in 1927 [7], by Fryze in 1931 [9], by Shepherd and Zakikhani in 1972 [18], by Kusters and Moore in 1980 [19], by Nabae and Akagi in 1984 [22], by Depenbrock in 1993 [32] and by Tenti in 2003 [45]. There are also numerous attempts aimed at development of the power theory that cannot be classified as schools, such as, for example [5, 6, 81, 10, 11, 12, 13, 14, 15, 16, 17, 24, 28, 40, 42, 44, 50, 51, 54, 64]. Some important discussions [3, 11, 24, 25, 28, 33, 35, 40, 41, 44, 46, 49, 51, 55, 57, 59, 60, 61, 67] should be mentioned as well.

Unfortunately, in the presence of the supply voltage distortion all these power theories were not capable of describing power properties for even a simple RL load shown in Fig. 3.

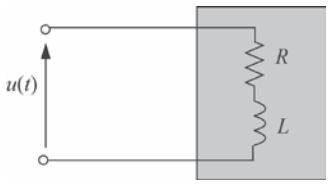

Fig. 3. RL load

Explanation of the energy flow phenomena in single- and in three-phase systems with nonsinusoidal voltages and currents was eventually provided in the frame of the Currents' Physical Components (CPC)-based power theory. A diagram which illustrates the development of the CPC power theory is shown in Fig. 4.

III. Currents' Physical Components: A Draft

The CPC concept is the first approach to power theory that has provided complete physical interpretation of power phenomena in electrical systems. It is also the first one that provided fundamentals for power factor improvement with reactive compensators at nonsinusoidal voltages and currents in single-phase, three-wire and four-wire systems. It also introduced a number of new concepts to the power theory such as the scattered current and power [21, 30], a load generated current [29], the unbalanced current and power, equivalent and unbalanced admittances [26] and the working active, reflected and detrimental powers [58].

The main steps towards the CPC-based power theory development were made by Fryze in [9], followed next by Shepherd and Zakikhani in [18]. Fryze introduced the concept of the ***active current*** $i_a(t)$, defined as

$$i_a(t) = \frac{P}{\|u\|^2} u(t) = G_e\, u(t) \tag{1}$$

where symbol $\|u\|$ denotes the voltage rms value. Shepherd and Zakikhani introduced in [18] the concept of the ***reactive current*** $i_r(t)$. When the supply voltage has harmonics of the order n from a set N and the Fourier series in a complex form

$$u(t) = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n e^{jn\omega_1 t}, \quad \boldsymbol{U}_n = U_n e^{j\alpha_n} \tag{2}$$

then the reactive current is defined as

$$i_r(t) \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_n \boldsymbol{U}_n e^{jn\omega_1 t} \tag{3}$$

where B_n is the load susceptance for the nth order harmonic.

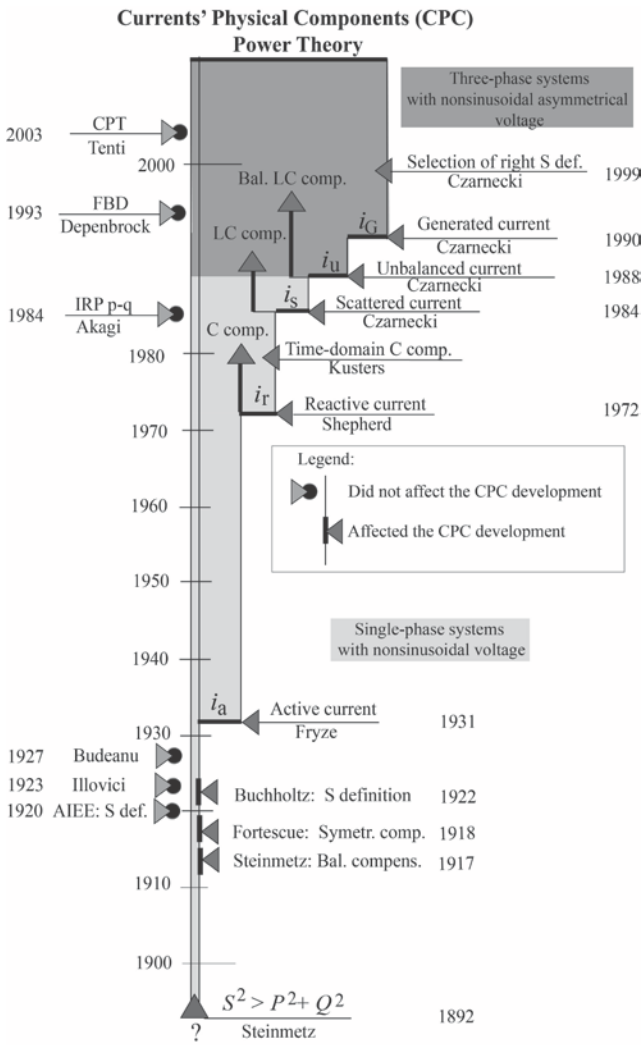

Fig. 4. Diagram which illustrates the development of the CPC-based power theory

The presence of the active and reactive currents in the supply current of a load is not sufficient for describing power properties of the load shown in Fig. 3, however. Such description became possible in 1984 when a new phenomenon in LTI loads, namely, an increase of the load current because of the change of the load conductance G_n with harmonic order, was discovered. The concept of the scattered current i_s was next introduced [21] to the power theory. It was defined as

$$i_s(t) \overset{df}{=} (G_0 - G_e)U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N}(G_n - G_e)\boldsymbol{U}_n\, e^{jn\omega_1 t}\ . \quad (4)$$

Consequently, the supply current of LTI loads supplied with a nonsinusoidal voltage can be decomposed into three components

$$i(t) = i_a(t) + i_s(t) + i_r(t)\,. \quad (5)$$

These components are associated with three different phenomena in the load and therefore they are referred to as Current Physical Components (CPC). They are mutually orthogonal, so that their rms values satisfy the relationship

$$\|i\|^2 = \|i_a\|^2 + \|i_s\|^2 + \|i_r\|^2\,. \quad (6)$$

It means that these phenomena affect the supply current rms value independently of each other. This enables the development of the CPC-based power equation of LTI loads supplied with a nonsinusoidal voltage

$$S^2 = P^2 + Q^2 + D_s^2 \quad (7)$$

where

$$Q \overset{df}{=} \|i_r\|\|u\| \quad (8)$$

is the ***reactive power***, while

$$D_s \overset{df}{=} \|i_s\|\|u\| \quad (9)$$

is the ***scattered power***.

Introduction of the concept of the scattered current to the power theory initiated the development of the Currents' Physical Components (CPC) power theory. It is based on the concept of the load current decomposition into components associated with distinctive phenomena in the load.

In the Steinmetz experiment the supply voltage was sinusoidal, while the current distortion was caused by harmonics generated by the load; however the quest for power definitions for a long time did not follow that paradigm. It was a quest for powers definitions even for linear loads with nonsinusoidal supply voltage.

The question of how to write a power equation of a current harmonics generating loads (HGLs) was raised [29] in 1990. The problem was illustrated with the circuit shown in Fig. 5

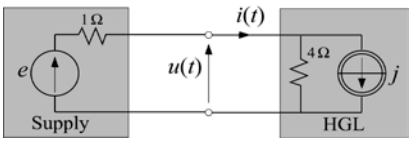

Fig. 5. Circuit with harmonic generating load (HGL).

with the internal voltage of the supply source

$$e = e_1 = 100\sqrt{2}\,\sin\omega_1 t \quad \mathrm{V}$$

and the load generated current

$$j = j_3 = 50\sqrt{2}\,\sin 3\omega_1 t \quad \mathrm{A}\,.$$

The voltage and current at the load terminals are equal to

$$u = u_1 + u_3 = 80\sqrt{2}\sin\omega_1 t - 40\sqrt{2}\sin 3\omega_1 t \quad \mathrm{V}$$

$$i = i_1 + i_3 = 20\sqrt{2}\sin\omega_1 t + 40\sqrt{2}\sin 3\omega_1 t \quad \mathrm{A}$$

thus, the active power is zero, since

$$P = \frac{1}{T}\int_0^T u\,i\,dt = \sum_{n=1,3} U_n\, I_n \cos\varphi_n = 1600 - 1600 = 0\,.$$

Consequently, in such a circuit

$$i_a(t) \equiv 0, \quad i_r(t) \equiv 0, \quad i_s(t) \equiv 0$$

thus how to write the power equation of such a load?

This circuit reveals a new power related phenomenon in systems with harmonic generating load (HGLs), namely, the active power of individual harmonics can be negative, which means that the energy at harmonic frequencies can flow from the HGL back to the supply source. The load current component composed of harmonics for which the active power P_n is negative, was referred to as the ***generated current***, i_G.

Decomposition of the load current into the active, reactive, scattered and generated currents has provided physical interpretation of power related phenomena in single-phase systems with nonsinusoidal voltages and currents and for compensation in such systems.

Explanation of the power properties of three-phase systems with nonsinusoidal and asymmetrical voltages and currents and the development of the power equation of such systems was hindered for a long time by a confusion on the apparent power S definition even if voltages and currents are sinusoidal.

In 1920 a committee of the American Institute of Electrical Engineers (AIEE) introduced [4] two definitions of the apparent power S, namely

$$S = U_R I_R + U_S I_S + U_T I_T = S_A \quad (10)$$

referred to as the ***arithmetical apparent power*** and

$$S = \sqrt{P^2 + Q^2} = S_G \quad (11)$$

referred to as the ***geometrical apparent power***. These two definitions were disseminated by the IEEE Standard Dictionary of Electrical and Electronic Terms [39]. In 1922 Buchholz [5] defined the apparent power

$$S = \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2} = S_B \quad (12)$$

but it was not known in the electrical engineering community.

When the voltages and currents are sinusoidal and symmetrical then these three definitions result in the same numerical value of the apparent power S. It is enough that the load line currents are asymmetrical, and these three definitions are no longer equivalent. It is illustrated in Fig. 6.

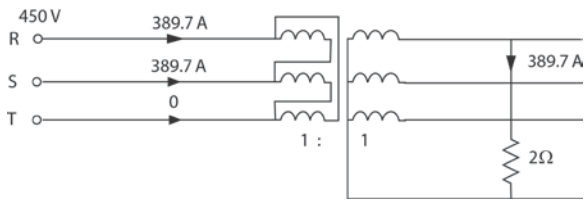

Fig. 6. Three-phase unbalanced system.

Assuming that the load is purely resistive and transformer is ideal, these three definitions result in the apparent power value

$$S_A = 351\ \mathrm{kVA}, \quad S_G = 304\ \mathrm{kVA}, \quad S_B = 430\ \mathrm{kVA}.$$

Consequently, depending on the selection of the apparent power definition, the power factor $\lambda = P/S$, is equal to

$$\lambda_A = \frac{P}{S_A} = 0.86, \quad \lambda_G = \frac{P}{S_G} = 1, \quad \lambda_B = \frac{P}{S_B} = 0.71.$$

Analysis of the apparent power definitions presented in [40], has demonstrated that the right value of the power factor λ is obtained only if the apparent power S is calculated according to the Buchholz' definition (10). Unfortunately, this definition in the circuit shown in Fig. 6 results in S = 430 kVA, while the active power P = 340 kW and the reactive power Q = 0, thus the commonly used power equation of three-phase systems:

$$S^2 = P^2 + Q^2$$

cannot be fulfilled. It does not describe power properties of three-phase unbalanced systems in a right way. A right equation is needed.

This equation in the frame of the CPC concept was found in [26] after a new current component associated with the load imbalance was introduced to the power theory. It is referred to as the ***unbalanced current*** $\boldsymbol{i}_u$. For LTI loads with a sinusoidal supply voltage

$$\boldsymbol{u}(t) = \begin{bmatrix} u_R(t) \\ u_S(t) \\ u_T(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\,\{\begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_S \\ \boldsymbol{U}_T \end{bmatrix} e^{j\omega t}\} = \sqrt{2}\,\mathrm{Re}\,\{\boldsymbol{U}\, e^{j\omega t}\} \quad (13)$$

this current was defined as

$$\boldsymbol{i}_u(t) = \begin{bmatrix} i_{Ru}(t) \\ i_{Su}(t) \\ i_{Tu}(t) \end{bmatrix} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\,\{\boldsymbol{Y}_u\, \mathbf{1}^n \boldsymbol{U}_R\, e^{j\omega t}\}\,. \quad (14)$$

Symbol $\boldsymbol{Y}_u$ in this formula denotes the ***unbalanced admittance*** of the load, which for the load with the equivalent circuit shown in Fig. 7

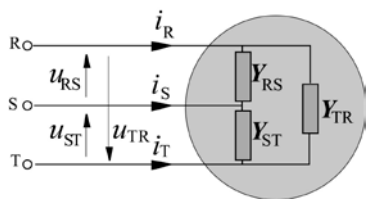

Fig. 7. Equivalent circuit of LTI three-phase load.

is equal to

$$\boldsymbol{Y}_u \overset{\mathrm{df}}{=} -(\boldsymbol{Y}_{ST} + \alpha\, \boldsymbol{Y}_{TR} + \alpha^* \boldsymbol{Y}_{RS}), \qquad \alpha = 1e^{j2\pi/3}\,. \quad (15)$$

Symbol $\mathbf{1}^n$ denotes a symmetrical unit vector of the negative sequence

$$\mathbf{1}^n \overset{\mathrm{df}}{=} \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{j2\pi/3} \\ 1e^{-j2\pi/3} \end{bmatrix}. \quad (16)$$

The CPC-based decomposition of the supply current of LTI loads supplied with sinusoidal and symmetrical voltage has the form

$$\boldsymbol{i}(t) = \boldsymbol{i}_a(t) + \boldsymbol{i}_r(t) + \boldsymbol{i}_u(t). \quad (17)$$

where

$$\boldsymbol{i}_a(t) = G_e\, \boldsymbol{u}(t) \quad (18)$$

is the ***active current***,

$$\boldsymbol{i}_r(t) = \sqrt{2}\,\mathrm{Re}\{\, jB_e \boldsymbol{U}\, e^{j\omega_1 t}\} \quad (19)$$

is a ***reactive current.***

Power equation of LTI unbalanced load supplied with sinusoidal and symmetrical voltage in the frame of the CPC –based power theory has the form

$$S^2 = P^2 + Q^2 + D_u^2 \quad (20)$$

where

$$D_u \overset{\mathrm{df}}{=} \|\boldsymbol{i}_s\|\,\|\boldsymbol{u}\| \quad (21)$$

is the ***unbalanced power***. The symbol

$$\|\boldsymbol{x}\| = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{x}^T(t)\,\boldsymbol{x}(t)\,dt} \quad (22)$$

in this formula denotes a ***three-phase rms value*** of a three-phase vector $\boldsymbol{x}$.

A CPC-based decomposition of the supply current of a three-phase load supplied with nonsinusoidal voltage [26] results in much more complex formulae than such decomposition in a sinusoidal situation. To reduce this complexity more compact symbols are needed. First, let us define a ***generalized unit three-phase vector***

$$\mathbf{1}_n = \begin{bmatrix} 1 \\ 1e^{-jn\frac{2\pi}{3}} \\ 1e^{jn\frac{2\pi}{3}} \end{bmatrix} = \begin{cases} \mathbf{1}^p, & \text{for } n = 3k+1 \\ \mathbf{1}^n, & \text{for } n = 3k-1\,. \\ \mathbf{1}^z, & \text{for } n = 3k \end{cases} \quad (23)$$

This symbol enables presentation of the supply voltage, composed of harmonics of the order n from a set N without zero sequence harmonics, i.e., of the order $n = 3k$, as

$$\boldsymbol{u}(t) = \begin{bmatrix} u_R(t) \\ u_S(t) \\ u_T(t) \end{bmatrix} = \sum_{n\in N} \boldsymbol{u}_n(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \mathbf{1}_n \boldsymbol{U}_{Rn}\, e^{jn\omega_1 t}\,. \quad (24)$$

The supply current of the load can be presented in the three-phase vector form

$$\boldsymbol{i}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{I}_{Rn} \\ \boldsymbol{I}_{Sn} \\ \boldsymbol{I}_{Tn} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_n e^{jn\omega_1 t} \quad (25)$$

and can be decomposed into Physical Components as follows:

$$\boldsymbol{i}(t) = \boldsymbol{i}_a(t) + \boldsymbol{i}_s(t) + \boldsymbol{i}_r(t) + \boldsymbol{i}_u(t). \quad (26)$$

In this decomposition

$$\boldsymbol{i}_a(t) = G_e\, \boldsymbol{u}(t) \quad (27)$$

is the ***active current***,

$$\boldsymbol{i}_s(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (G_{en} - G_e)\, \mathbf{1}_n \boldsymbol{U}_{Rn}\, e^{jn\omega_1 t} \quad (28)$$

is a ***scattered current***,

$$\boldsymbol{i}_r(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_{en}\, \mathbf{1}_n \boldsymbol{U}_{Rn}\, e^{jn\omega_1 t} \quad (29)$$

is a ***reactive current*** and

$$\boldsymbol{i}_{\mathrm{u}}(t) = \sqrt{2}\ \mathrm{Re} \sum_{n \in N} \boldsymbol{Y}_{\mathrm{u}n} \mathbf{1}_n^* \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{30}$$

with

$$\boldsymbol{Y}_{\mathrm{u}n} = -\left(\boldsymbol{Y}_{\mathrm{ST}n} + e^{jn\frac{2\pi}{3}} \boldsymbol{Y}_{\mathrm{TR}n} + e^{-jn\frac{2\pi}{3}} \boldsymbol{Y}_{\mathrm{RS}n}\right) \tag{31}$$

is an ***unbalanced current.***

Each of these currents is associated with a distinctive physical phenomenon in the load. They are mutually orthogonal, so that their three-phase rms values satisfy the relationship

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_{\mathrm{a}}\|^2 + \|\boldsymbol{i}_{\mathrm{s}}\|^2 + \|\boldsymbol{i}_{\mathrm{r}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}\|^2 . \tag{32}$$

The CPC-based power equation has the form

$$S^2 = P^2 + D_{\mathrm{s}}^2 + Q^2 + D_{\mathrm{u}}^2 . \tag{33}$$

Presentation of the whole CPC-based power theory is beyond the scope of this paper, however. This chapter is confined to presentation of only the main concept and fundamentals of the CPC. It presently provides interpretation of power phenomena, power definitions and equations, as well as fundamentals of reactive and switching compensation for single-phase, three-wire and four-wire systems with linear, time-invariant and harmonics generating loads. The CPC – based power decomposition in three-phase systems with asymmetrical nonsinusoidal supply voltage is presented in [65], while the power properties of four-wire systems with nonsinusoidal, but symmetrical voltage is presented in [62].

The CPC- based power theory was originally developed for systems with periodic voltages and currents, but in [43] and [63], it was extended for systems where voltages and currents are semi-periodic.

IV. Approaches to Power Theory Development

Development of the CPC was only one among numerous approaches to the power theory. The debate on definitions of powers in systems with nonsinusoidal voltages and currents, often focused on the reactive power definition, seems to be one of the most tenacious debates in electrical engineering. It was even the subject of special conferences run periodically, such as the International Workshop on the Reactive Power and Measurements, run in Italy, or the International School on Nonsinusoidal Currents and Compensation (ISNCC) run bi-annually in Poland.

The main difference in various approaches to the power theory development boils down to two questions:

- *Should the power properties of a system be described in the **time-domain** or in the **frequency-domain**?*
- *Should the power properties of a system be described in terms of **instantaneous** or in terms of **averaged** over the period values?*

Let us discuss the first question. Mathematically, due to Fourier Transforms, descriptions of continuous quantities in the time-domain and in the frequency-domain are mutually equivalent. The question whether the power theory should be formulated in a frequency or in a time-domain, as suggested in Refs. [9, 23, 32], or not boils down to the question: *should the concept of harmonics be used for that purpose or not?*

To answer such a question two aspects of the issue should be taken into account. The first of them is metrological availability of harmonics. Such availability can be crucial for technical implementations of a harmonics-based power theory. When the concept of harmonics was introduced to the power theory by Illovici [6], and Budeanu [7], it was possible to measure, using tuned filters, only the rms value of harmonics. Their phase was practically beyond measurement possibility. Now, sampling and digital signal processing (DSP) are capable of providing complex rms values of harmonics up to relatively high order in real time. Thus availability is not an issue.

The second aspect of the harmonics issue is more crucial: does the concept of harmonics contribute to our comprehension of power phenomena or hinder it?

Thinking of voltages and currents as sums of harmonics when applied to the energy flow studies may lead to unacceptable conclusions. This was demonstrated by Fryze in [9] who analyzed the energy flow in the circuit shown in Fig. 8, with dc supply voltage and a periodic switch.

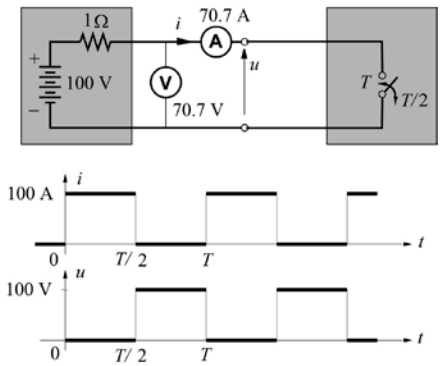

Fig. 8. Circuit with a periodic switch.

According to the frequency-domain approach, the load voltage and current can be expressed in terms of harmonics in the forms

$$u(t) = U_0 + \sqrt{2} \sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n) = \sum_{n=0}^{\infty} u_n \tag{34}$$

$$i(t) = I_0 + \sqrt{2} \sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \beta_n) = \sum_{n=0}^{\infty} i_n . \tag{35}$$

The instantaneous power $p(t)$ at the load terminals, i.e., the rate of energy, $W(t)$, that flows from the supply source to the load is equal to

$$p(t) = \frac{dW}{dt} = u(t)\, i(t) = \sum_{r=0}^{\infty} u_r \sum_{s=0}^{\infty} i_s = \sum_{n=0}^{\infty} S_n \cos(n\omega_1 t + \psi_n) . \tag{36}$$

Thus, it is equal to an infinite sum of oscillating components.

This formula suggests the existence of an infinite number of oscillating components in the instantaneous power. To find the instantaneous power at any instant of time, this series has to be calculated.

In the time-domain it is directly visible from the voltage and current waveforms that, apart from discontinuity points, their product

$$p(t) = u(t)\, i(t) \equiv 0 \ . \tag{37}$$

At discontinuity points a tiny amount of energy is only stored in a stray inductance and capacitance of conductors.

Thus there are no flow of energy and no energy oscillation in such a circuit. The time-domain approach is clearly superior to the frequency-domain in such a situation. There are quite opposite situations, however.

The power theory of single-phase circuits with LTI loads at nonsinusoidal supply voltage was formulated eventually [21] using the CPC approach with Shepherd and Zakikhani's (S&Z) contribution [18] in the frequency-domain. The reactive current $i_r(t)$ and the scattered current $i_s(t)$ are defined in the frequency-domain. The first is associated with the phase-shift of the voltage and current harmonics, the second is associated with the change of the load conductance with the harmonic order.

The presence of these two currents in the supply current is the cause for which the apparent power S is higher than the load active power P. Thus the answer to the main questions of the power theory was not provided in the time-domain, but in the frequency-domain. Equivalent decomposition in the time-domain that would reveal the cause of the apparent power increase is not known.

Because the time and frequency-domains are mutually equivalent, the scattered current, originally identified in the frequency-domain, can be also somehow recalculated and expressed in the time-domain. The physical interpretation of this current's existence, namely, as the effect of a change of the load conductance with harmonic frequency cannot be used in the time-domain, however.

Let us discuss the second question related to instantaneous versus averaged approach.

The most fundamental power quantity in electrical systems, the instantaneous power $p(t)$, specifies the rate of the electric energy flow at each instant of time. Due to this interpretation it is the most unquestionable power quantity. This can imply a conclusion that the whole power theory should be based on quantities defined as instantaneous ones.

There are situations in power systems where indeed instantaneous values of voltages and currents are crucial, as it is during disturbances or faults. Performance of electrical systems with periodic voltages and currents is specified at normal operation entirely in terms of quantities defined as some integrals over the supply voltage period T, however. These are the active, reactive and apparent powers, the voltage and current rms value, the power factor, rms value of harmonics, harmonic distortion, or the voltage and current symmetrical components. The instantaneous power $p(t)$ usually is not a matter of interest for system designers and operators. In fact, terms like "*rms*", "*apparent power*", "*harmonic*" and "*symmetrical component*", are alien for theories that describe instantaneous properties of electrical systems.

The major difficulty of power theories that claim to be "instantaneous" stems from the fact that power properties of the load cannot be identified instantaneously. This is illustrated in Fig. 9, with an unknown load and a pair of instantaneous values of the load voltage and current. As shown in Fig. 10, in the "black box" could be a resistor, an inductor or a capacitor.

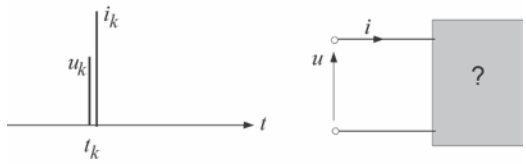

Fig. 9. Unknown load and pair of instantaneous values of the load voltage and current.

In fact, an infinite number of different loads can have identical pairs of voltage and current samples. Taking into account that the load voltage and current are in general nonsinusoidal, the instantaneous values of the voltage and current over the whole period T have to be measured to draw conclusions on the load power properties.

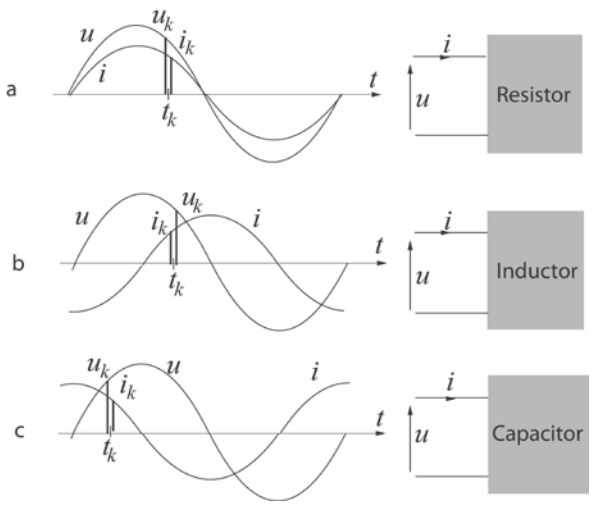

Fig. 10. Different loads with identical pairs of the voltage and current instantaneous value.

Quantities obtained by averaging could be added to the instantaneous theory, but this would undermine the claim that the theory is indeed instantaneous. It is just the case of the Instantaneous Reactive Power *p-q* Theory, where the active power P can be found only by averaging the instantaneous active power p, but even with this averaging, the concepts of the apparent power S and consequently, the concept of the power factor does not exist inside of this theory.

V. Comments on Some Power Theories

Budeanu's power theory formulated in 1927 [7] in the frequency domain, was the first major attempt aimed at description of power properties of electrical loads supplied with nonsinusoidal voltage. It is probably the most widely distributed power theory of single-phase circuits with nonsinusoidal voltages and currents. Correctness of this theory was challenged in 1987 [25], where it was demonstrated that

- There is no physical phenomenon associated with the Budeanu's reactive power Q.
- There is no association between Budeanu's distortion power D and the voltage and current mutual distortion.
- There is no relation between the power factor improvement and the Budeanu's reactive power reduction.

Fryze's power theory, formulated in 1931 [9] in the time-domain, introduced an important concept of the active current and the load current decomposition into orthogonal components. It introduced the concept of the reactive

current, but did not provide its physical interpretation other than that it is a useless current.

Shepherd and Zakikhani's power theory formulated in 1972 in the frequency-domain in [18] provided a right definition of the reactive current $i_r(t)$. It raised and solved the issue of calculation of the optimal capacitance C_{opt}, which in the presence of the supply voltage harmonics increases the power factor to the maximum possible value. Unfortunately, the active power P was lost in the power equation they developed.

Kuster and Moore's power theory formulated in 1980 [19] in the time-domain, solved the problem of the optimal capacitance calculation without using the concept of harmonics. The solution was valid, as shown in [20], only at ideal supply source, however.

Instantaneous Reactive Power p-q Theory, formulated by Nabae, Akagi and Kanazawa in 1984 [22], in the time-domain, and generalized in [40] provides fundamentals for switching compensator control. This control algorithm is valid only at sinusoidal [53] and symmetrical [56] supply voltage, however. Moreover, the IRP p-q Theory misinterprets the power phenomena [46, 60] in electrical systems.

FBD Method developed by Depenbrock [32] in the time-domain, is a generalization of Fryze's power theory for three-phase systems. It correctly defines, after Buchholtz [5], the apparent power S, but has all the disadvantages of Fryze's power theory.

The Conservative Power Theory (CPT), developed by Tenti and others in [45], is formulated in the time-domain and has strong analogy to Budeanu's power theory and its deficiencies, although it follows Fryze's concept of the current orthogonal decomposition. The current components in the CPT do not have physical interpretation, however. Moreover the CPT does not provide, as shown in [61, 67], right fundamentals for a capacitive compensator design. It also wrongly defines, as shown in [66], the unbalanced power in three-phase systems.

VI. Powers in Three-Phase Systems

Since most of the electric energy is transferred by three-phase systems, power properties of such systems are of the crucial importance for the power systems economy. Therefore, there was a lot of research [2, 3, 8, 10, 11, 12, 13, 15, 17, 24, 33, 51, 54, 64] focused on the development of the power theory of three-phase systems in the presence of harmonics and asymmetry.

There were two major obstacles which made these efforts doomed to fail. First, three-phase systems have as their sub-set all single-phase systems. As long as the power properties of single-phase systems were not correctly described and comprehended, it was not possible to correctly describe such properties of three-phase systems. Second, all studies were carried on using a wrong definition of the apparent power S of three-phase systems. In 1920 a committee of AIEE suggested [4] two definitions of the apparent power S, known as the arithmetical definition and the geometrical definition of that power. A debate [8, 10, 11] on these two definitions in the twenties was inconclusive and both of them were eventually adopted as standard definitions. They can be found in the IEEE Standard Dictionary of Electrical and Electronics Terms [39]. There was also a definition of the apparent power suggested in 1922 [5] by Buchholtz, but it was dominated by the arithmetic and the geometric definitions, endorsed by the IEEE, and it was not used.

When voltages and currents are sinusoidal and symmetrical these three definitions result in the same numerical value of the apparent power S and consequently, the power factor λ. When this condition is not satisfied, the value of the power factor λ depends on the apparent definition.

The issue of which definition of the apparent power provides the right value of the power factor was investigated in 1999 and results presented in [41]. It was demonstrated there that in the presence of current asymmetry the arithmetic and the geometric definitions result in a wrong value of the power factor. Right value was obtained when the apparent power S was calculated according to Buchholz' definition. This selection of the right definition of the apparent power S eventually paved the road for describing unbalanced three-phase systems, in the frame of the CPC, in power terms.

VII. Compensation

Compensation in systems with nonsinusoidal voltages and currents is the major practical implementation of the power theory of such systems. It can be divided into three groups:

- Reactive compensation.
- Switching compensation.
- Hybrid, i.e., reactive and switching compensation.

Observe, however, that in the case of very high power loads, for example, arc furnaces, switching compensation cannot be used because of limited switching power of transistors needed for such compensator construction. Such loads can be compensated only by reactive compensators.

The progress in the development of methods of compensation followed, of course, the progress in the development of the power theory.

The first result on compensation was obtained by Shepherd and Zakikhani [18] who found in 1972 a method of calculation of an optimal capacitance C_{opt} which increases the power factor to the maximum value. This result was obtained in the frequency-domain. In 1980 a method of calculation of the same optimum capacitance, but in the time-domain, was suggested by Kusters and Moore [19]. Unfortunately, in common industrial situations where the supply sources have some inductive impedance, the effect of the compensating capacitor upon the supply voltage may not allow [20] for the optimal capacitance calculation as suggested in [19]. The same applies to the method developed by Shepherd and Zakikhani. Capacitive compensation in systems with nonsinusoidal voltages and currents causes increase in the waveform distortion and consequently, such compensation should be avoided.

Fundamentals of total compensation of the reactive current by a shunt reactive compensator were developed in [21], where it was also shown that the scattered current cannot by compensated by such a compensator. A series-shunt compensator is needed [30] for the scattered current compensation.

Control fundamentals for switching compensators (SC) are the frame of the Instantaneous Reactive Power (IRP) p-q Theory in [22]. The FBD Method [32] and the CPC-based algorithms [47, 48, 52, 55] can also be used for that purpose. Unfortunately, when the supply voltage contains harmonics or is asymmetrical then the IRP-based algorithm

does not provide [53, 56, 60] right control for the compensator. It works properly only at symmetrical and sinusoidal supply voltages.

The CPC-based control algorithms have two major advantages. Such algorithms can provide control both for the SC and the reactive compensators, i.e., for hybrid compensators. Moreover, it is up to the designer's discretion which harmful component of the supply current should be compensated. In a presence of limited resources, in particular the compensator limited power, it enables flexible compensation.

Compensation of unbalanced three-phase loads requires, apart from compensation of the load generated harmonics, that not only the reactive, but also the unbalanced current be reduced.

Calculation of LC parameters of a reactive compensator can be solved as an optimization problem, without any power theory. Such a specific solution has very limited contribution to the knowledge on compensation, however. Such optimization is usually a time consuming process, inappropriate for adaptive compensation of fast varying loads. Algebraic formulae that specify compensator LC parameters are more convenient for that.

The first reactive balancing compensator for systems with sinusoidal voltages and currents was developed [2] by Steinmetz in 1917. It is known as the Steinmetz circuit.

A method of calculation of LC parameters of a balancing compensator for total compensation in systems with nonsinusoidal supply voltage was developed in [27]. A method of minimization of the reactive and unbalanced current with a reduced complexity compensator was presented in [31].

VIII. Conclusions

The Currents' Physical Components – based power theory was developed with the main focus on providing solid physical fundamentals for description of the energy flow phenomena in electrical systems as well as on providing reliable fundamentals for compensation in such systems. The approach has been successful. The CPC-based power theory explains and describes all phenomena associated with the energy flow in systems with nonsinusoidal and asymmetrical voltages and currents. It is now the only approach to power theory that provides fundamentals for compensation both with reactive and with switching compensators. The CPC-based compensation is founded not on a vague engineering intuition but on clear physical fundamentals.

The research focus can be shifted now towards implementations of the CPC – based power theory in systems where the voltage distortion and asymmetry should be taken into account at such systems' economic optimization. These are, first of all, systems with very high power arc furnaces [66, 68, 69]. Islanded micro-grids may also belong to this category, due to high saturation with power electronics equipment and relatively low short-circuit power. Consequently, the voltage distortion and asymmetry in micro-grids could be higher than in common distribution systems.

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